TI in Focus: AP® Calculus

2020 Mock AP® Calculus Exam

AB-2: Solutions, Concepts, and Scoring Guidelines

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AB 2

t	0	2	6	8	10	12
$y'(t)$	4	8	-2	3	-1	-5

The vertical position of a particle moving along the y-axis is modeled by a twice-differentiable function $y(t)$ where t is measured in seconds and $y(t)$ is measured in meters.

Selected values of $y'(t)$, the derivative of $y(t)$, over the interval $0 \leq t \leq 12$ seconds are shown in the table above.

The position of the particle at time $t = 12$ is $y(12) = -3$.

(c) Using correct units, explain the meaning of $y''(4)$ in the context of the problem.

Key Concepts

Derivatives and Units

Use Leibniz notation to determine the units associated with a derivative.

$\frac{dy}{dx}$: the units of y divided by the units of x.

Example:

Suppose y is measures in miles and x is measured in hours.

$\frac{dy}{dx}$: units are miles per hour

$\frac{d^2y}{dx^2} = \frac{dy'}{dx}$: units are (miles per hour) per hour, or miles per hour.^{2}

Solution

y is measured in meters and t is measured in seconds.

$y'(t) = \dfrac{dy}{dt}$: units are meters per second

Velocity: the change in position over time

$y''(t) = \dfrac{dy'}{dt} = \dfrac{d^2y}{dt^2}$: units are (meters per second) per second, or meters per second^2

Acceleration: the change in velocity over time

Therefore $y''(4) = \left.\dfrac{d^2y}{dt^2}\right|_{t=4}$:

the acceleration of the particle in meters per second per second at time $t = 4$ seconds.

Scoring Guidelines

(c) $y''(4)$ is the acceleration of the particle in meters per second per second at time $t = 4$ seconds.	1 : meaning with units

Scoring Notes

1: meaning with units

- Responses that earn the point:
  - $y''(4)$ is the acceleration of the particle at $t = 4$ in m/s^2
  - $y''(4)$ is the rate of change of the particle's velocity at $t = 4$ in m/s^2
  - The particle is accelerating at a rate of $y''(4)$ m/s^2 at $t = 4$.
- Responses that do not earn the point:
  - $y''(4)$ is the rate of change of speed at $t = 4$ in m/s^2
  - $y''(4)$ means the particle is speeding up at $t = 4$ in m/s^2
- Units must include the rate of change units, but do no need to include t (time) units. However, if t units are given, they must be correct.
- Incorrect units presented override any units presented in parts (a) or (b).
- Read $x = 4$ as $t = 4$

(d) Find the average value of the acceleration of the particle over the interval $[0, 12]$.

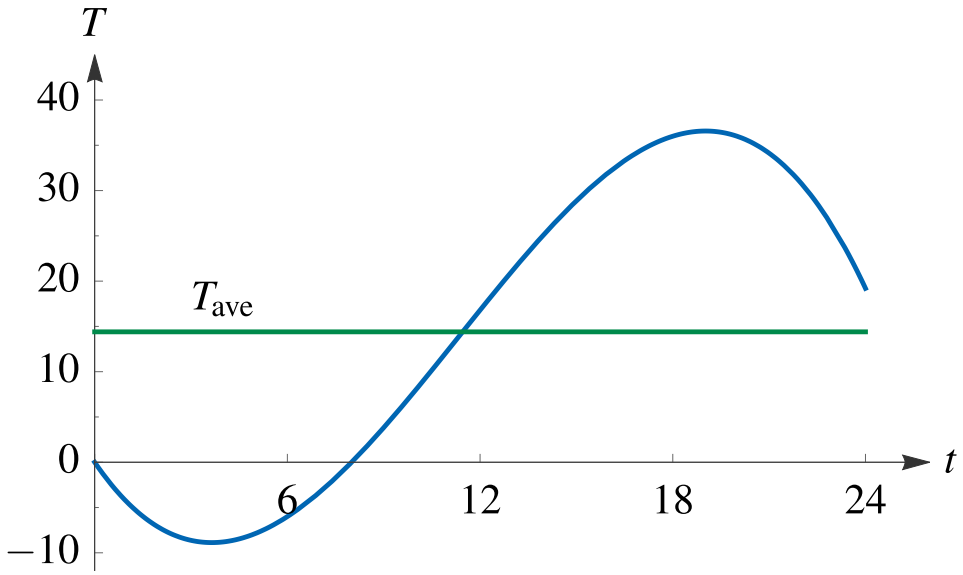
Key Concepts

Average Value of a Function

Arithmetic mean of a finite set of numbers $y_1, y_2, \ldots, y_n$:

$$y_{\text{ave}} = \frac{y_1 + y_2 + \cdots + y_n}{n}$$

Example: the average temperature during a day.



Average Value of a Function

Consider the average value of a function over the interval $[a, b]$.

Construct a regular partition of $[a, b]$: $\{a = x_0, x_1, x_2, \ldots, x_n = b\}$, $\Delta x = \dfrac{b-a}{n}$

Select values $x_i^* \in [x_{i-1}, x_i]$

Compute the average: $\dfrac{f(x_1^*) + f(x_2^*) + \cdots + f(x_n^*)}{n}$

Since $\Delta x = \dfrac{b-a}{n} \quad \Rightarrow \quad n = \dfrac{b-a}{\Delta x}$

$$\frac{f(x_1^*) + f(x_2^*) + \cdots + f(x_n^*)}{\dfrac{b-a}{\Delta x}} = \cdots = \frac{1}{b-a} \sum_{i=1}^{n} f(x_i^*)\, \Delta x$$

Average Value of a Function

This summation is a Riemann sum and the limiting value is the definition of a definite integral.

$$\lim_{n\to\infty} \frac{1}{b-a} \sum_{i=1}^{n} f(x_i^*)\,\Delta x = \frac{1}{b-a} \int_a^b f(x)\,dx$$

The **average value of f** on the interval $[a, b]$ is defined to be

$$f_{\text{ave}} = \frac{1}{b-a} \int_a^b f(x)\,dx$$

If $f(x) \geq 0$ on the interval $[a, b]$, then

$$\text{average height} = \frac{\displaystyle\int_a^b f(x)\,dx}{b-a} = \frac{\text{area}}{\text{width}}$$

The Fundamental Theorem of Calculus, Part 2

If f is continuous on $[a, b]$, then

$$\int_a^b f(x)\,dx = F(b) - F(a)$$

where F is any antiderivative of f, that is, a function such that $F' = f$.

Solution

$$\text{Ave value} = \frac{1}{12-0}\int_0^{12} y''(t)\,dt$$ Average value definition

$$= \frac{1}{12}\Big[y'(t)\Big]_0^{12} = \frac{1}{12}[y'(12) - y'(0)]$$ FTC, part 2

$$= \frac{1}{12}[-5-4] = -\frac{3}{4}$$ Use values from the table

Scoring Guidelines

(d) $\frac{1}{12}\int_0^{12} y''(t)\,dt = \frac{1}{12}\Big[y'(t)\Big]_0^{12}$

$$= \frac{1}{12}[y'(12) - y'(0)]$$

$$= \frac{1}{12}[-5-4] = -\frac{3}{4}$$

$$3: \begin{cases} 1: \text{integral} \\ 1: \text{Fundamental Theorem of Calculus} \\ 1: \text{answer} \end{cases}$$

Scoring Notes

1: integral

- Earned for the integral expression with correct limits.
- Must address the average value of y''.
- The constant is associated with the answer point.
  An incorrect or missing constant will not earn the third point.

1: Fundamental Theorem of Calculus

- Earned for the expression $y'(12) - y'(0)$

1: answer

- Earned only for the correct answer.
- The values of -5 and 4 must be shown.
- $\frac{1}{12}(-5-4)$: earns two points, 0 - 1 - 1

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