TI in Focus: AP® Calculus

2020 Mock AP® Calculus Exam

AB-2: Solutions, Concepts, and Scoring Guidelines

Stephen Kokoska
Professor, Bloomsburg University
Former AP® Calculus Chief Reader

*AP is a registered trademark of the College Board, which was not involved in the production of, and does not endorse, this product.

AB 2

t	0	2	6	8	10	12
$y'(t)$	4	8	-2	3	-1	-5

The vertical position of a particle moving along the y-axis is modeled by a twice-differentiable function $y(t)$ where t is measured in seconds and $y(t)$ is measured in meters.

Selected values of $y'(t)$, the derivative of $y(t)$, over the interval $0 \leq t \leq 12$ seconds are shown in the table above.

The position of the particle at time $t = 12$ is $y(12) = -3$.

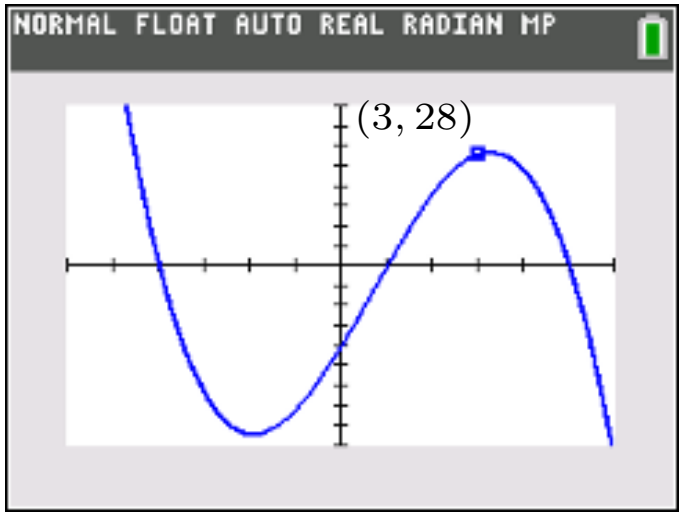
(a) Use a locally linear approximation of y at $t = 12$ to approximate $y(11.8)$.

Key Concepts

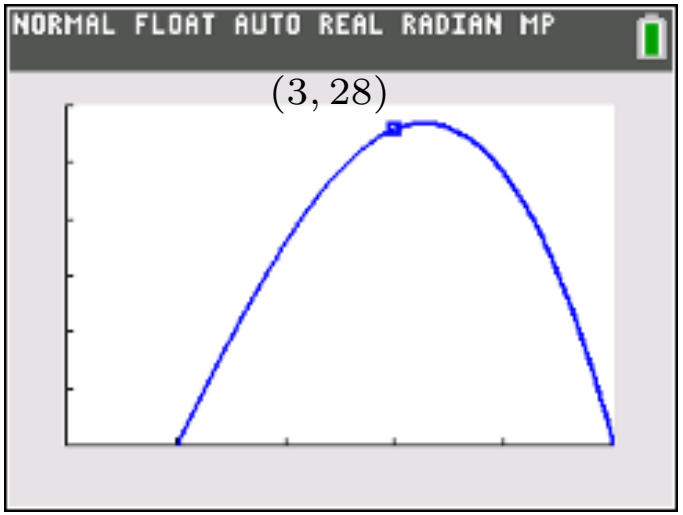
Local Linearity

- If f is differentiable at $(a, f(a))$, then the graph of f is smooth and has a tangent line at this point.
- There is no sharp edge or cusp; the graph of f does not change direction abruptly.
- Geometrically, if we zoom in near $(a, f(a))$ the graph of f straightens out.
- This geometric property: local linearity.

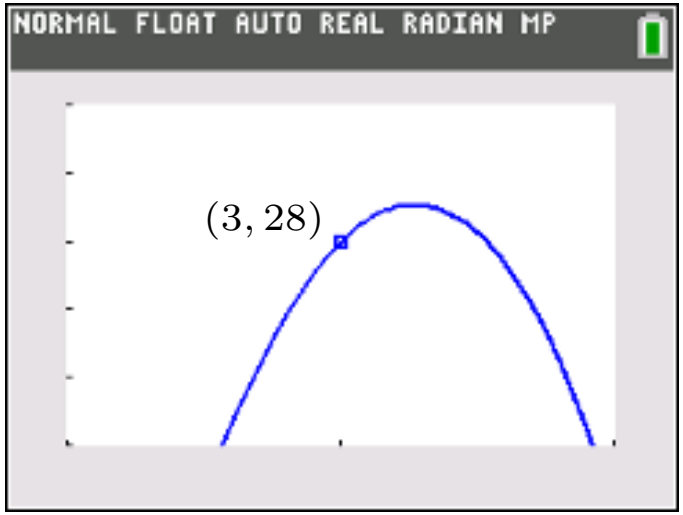
Consider the function $f(x) = -20 + 19x + 2x^2 - x^3$



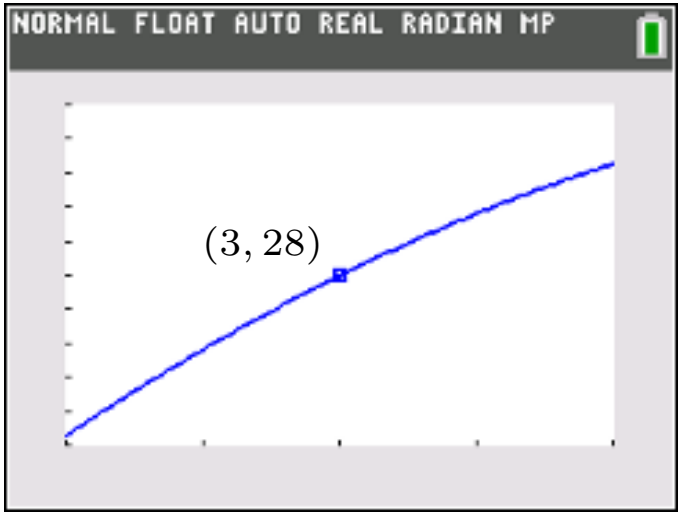
$[-6, 6] \times [-45, 40]$



$[0, 5] \times [20, 30]$



$[2, 4] \times [25, 30]$

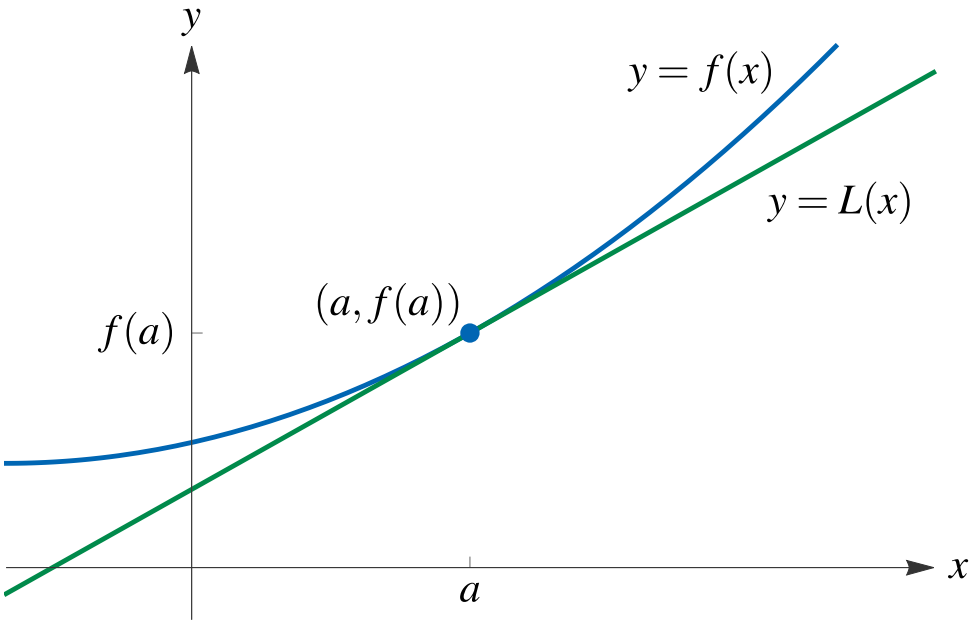


$[2.9, 3.1] \times [27.5, 28.5]$

Tangent Line Approximation

Suppose f is differentiable at $x = a$ and $f(a)$ is known.

Let L be the linear function whose graph is the tangent line to the graph of f at the point $(a, f(a))$.



An equation of the tangent line: $y = f(a) + f'(a)(x - a)$

Linear approximation or **tangent line approximation**:
$f(x) \approx f(a) + f'(a)(x - a)$

Linearization: $L(x) = f(a) + f'(a)(x - a)$

A Closer Look

The accuracy of the tangent line approximation depends on three factors:

1. The distance between x and a.
2. The steepness of the graph of f near a.
3. The concavity of the graph of f.

Solution

$y(12) = -3$ Given

$y'(12) = -5$ From the table

An equation of the tangent line:

$$y = y(12) + y'(12)(x - 12) = -3 - 5(x - 12)$$

Therefore

$$y(11.8) \approx -3 - 5(11.8 - 12) = -3 - 5(-0.2) = -2$$

Scoring Guidelines

(a) $y'(12) = -5$

An equation of the tangent line is $y = (-5)(x - 12) - 3$

$y(11.8) \approx (-5)(11.8 - 12) - 3 = -2$

$2: \begin{cases} 1 : \text{tangent line equation} \\ 1 : \text{approximation} \end{cases}$

Scoring Notes

1: tangent line equation

- Earned for the correct tangent line equation, in any form.
- No explanation or additional supporting work necessary.
- If error in rewriting: does not earn the second point.
- x instead of t OK.
- $y(x) =$ OK.

1: approximation

- $\approx$ not needed.
- $y(11.8) \approx -5(11.8 - 12) - 3 = -2$

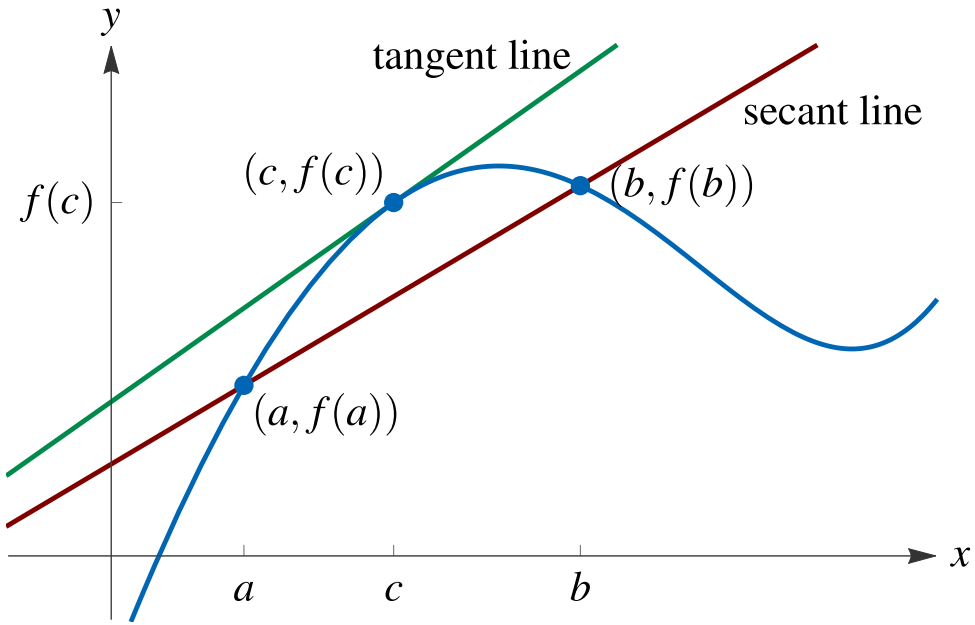
Texas Instruments

(b) Approximate $y''(4)$ using the average rate of change of $y'(t)$ on the interval $2 \leq t \leq 6$.

Key Concepts

Suppose f is differentiable at $x = c$ and $f(c)$ is known.

The slope of any secant line *close* to $x = c$ approximates the slope of the tangent line at $x = c$.



$$m_{\text{sec}} = \frac{f(b) - f(a)}{b - a} \approx f'(c)$$

Solution

t	0	2	6	8	10	12
$y'(t)$	4	8	-2	3	-1	-5

$$y''(4) \approx \frac{y'(6) - y'(2)}{6 - 2} = \frac{-2 - 8}{4} = -2.5$$

Scoring Guidelines

(b) $y''(4) \approx \dfrac{y'(6) - y'(2)}{6 - 2} = \dfrac{-2 - 8}{4} = -2.5$	1 : approximation

Scoring Notes

1: approximation

- Need to see a difference quotient: a difference *and* a quotient.
- Bald -2.5 does not earn the point.
- Must use values from the table.

education.ti.com