TI in Focus: AP® Calculus

2020 Mock AP® Calculus Exam

AB-1: Solutions, Concepts, and Scoring Guidelines

Stephen Kokoska
Professor, Bloomsburg University
Former AP® Calculus Chief Reader

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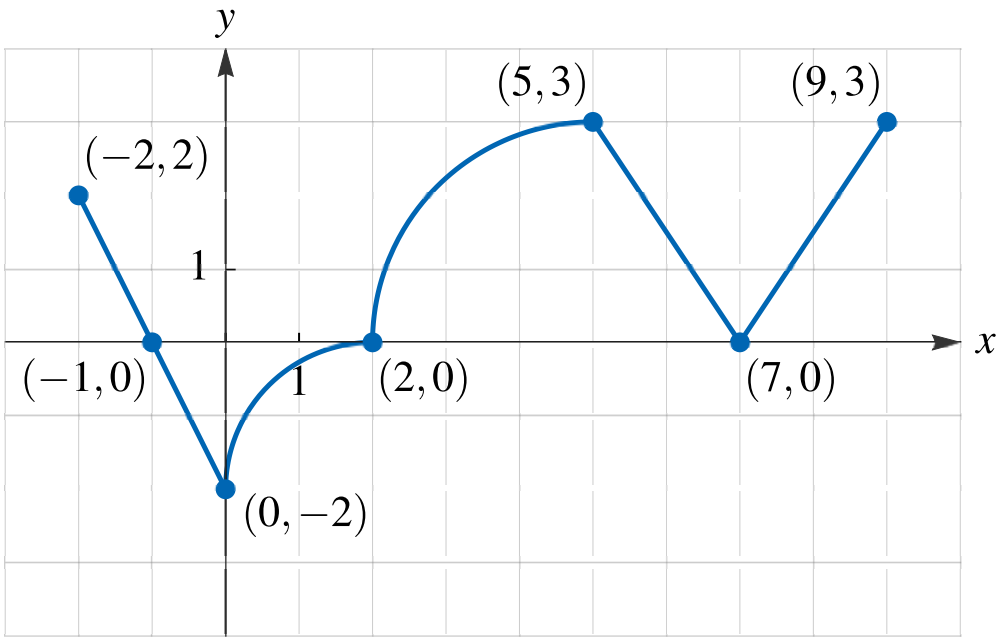
AB 1

The continuous function f has domain $-2 \leq x \leq 9$. The graph of f, consisting of three line segments and two quarter circles, is shown in the figure.



Graph of f

Let g be the function defined by $g(x) = \int_0^x f(t)\,dt$ for $-2 \leq x \leq 9$.

(f) Find the absolute maximum value of g over the interval $-2 \leq x \leq 5$.

Key Concepts

The Closed Interval, or Table of Values, Method

To find the absolute maximum and minimum value of a continuous function f on a closed interval $[a, b]$:

1. Find the values of f at the critical number of f in (a, b).
2. Find the values of f at the endpoints of the interval.
3. The largest of the values from Steps 1 and 2 is the absolute maximum value; the smallest of these values in the absolute minimum value.

Recall: The definite integral can be interpreted as net area.

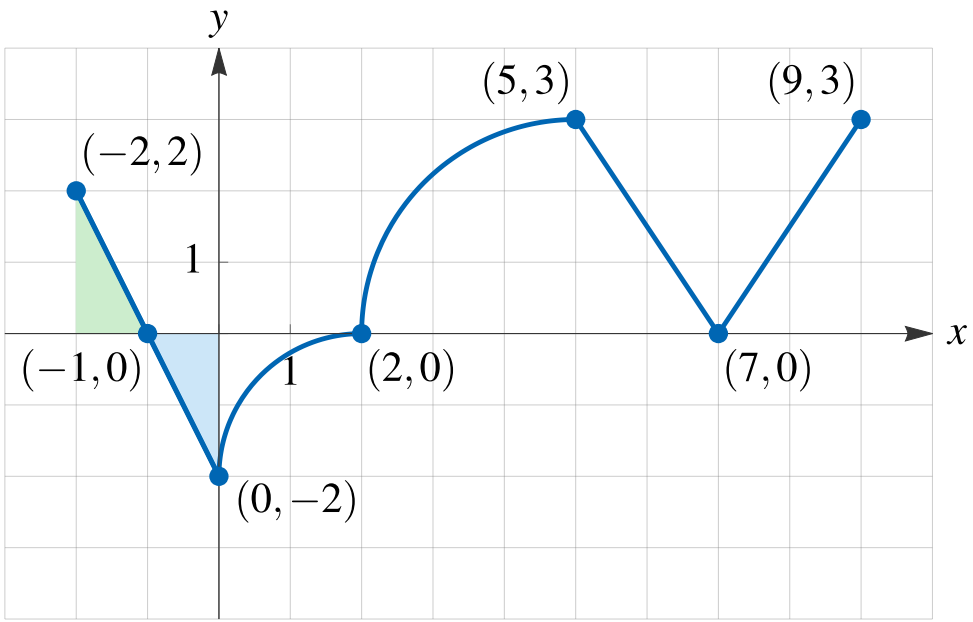
Solution

g is a continuous function on the closed interval $[-2, 5]$

Evaluate g at the endpoints of the interval and at the critical points in the interval $(-2, 5)$

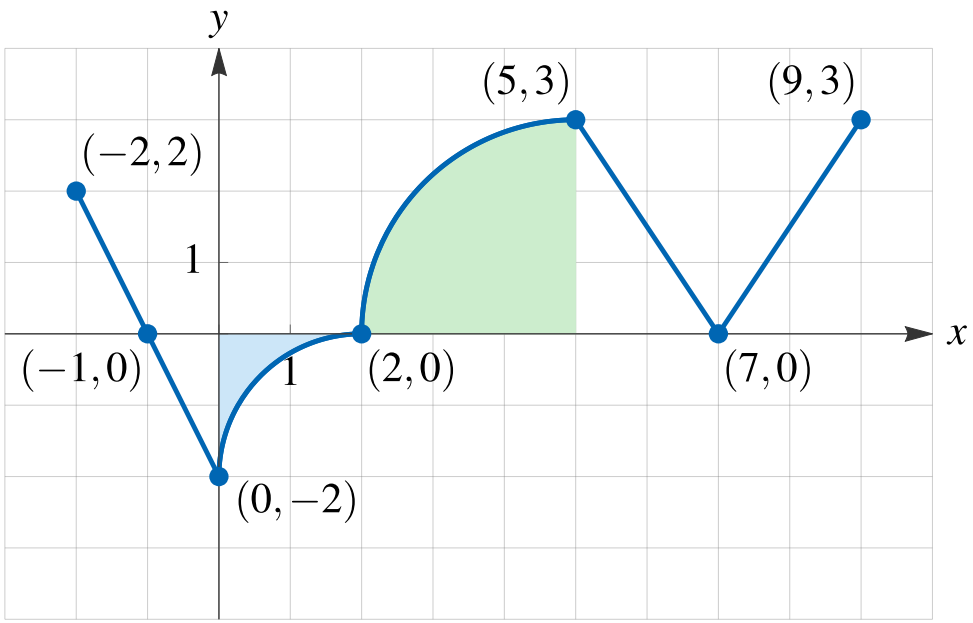
$g(-1) = 1$ from part (d)

$g(2) = \pi - 4$ from part (e)



$$g(-2) = \int_0^{-2} f(t)\,dt = -\int_{-2}^{0} f(t)\,dt = -\left(\frac{1}{2}(1)(2) - \frac{1}{2}(1)(2)\right) = 0$$

Solution (Continued)



$$g(5) = \int_0^5 f(t)\,dt = \int_0^2 f(t)\,dt + \int_2^5 f(t)\,dt$$

$$= (\pi - 4) + \frac{1}{4} \cdot \pi \cdot 3^2 = \frac{13}{4}\pi - 4$$

Solution (Continued)

x	$g(x)$
-2	0
-1	1
2	$\pi - 4$
5	$\frac{13}{4}\pi - 4 = 6.21$

The absolute maximum value of g is $\frac{13}{4}\pi - 4$.

Scoring Guidelines

(f) The absolute maximum value occurs at an endpoint of the interval or a critical point.

Consider a table of values.

x	$g(x)$
-2	0
-1	1
2	$\pi - 4$
5	$\pi - 4 + \frac{1}{4}\pi 3^2 = \frac{13}{4}\pi - 4$

The absolute maximum value of g is $\frac{13}{4}\pi - 4$.

$$4: \begin{cases} 1 : \text{considers } x = -2 \text{ and } x = 5 \\ 1 : \text{considers } x = -1 \text{ and } x = 2 \\ 1 : \text{answer} \\ 1 : \text{justification} \end{cases}$$

Scoring Notes

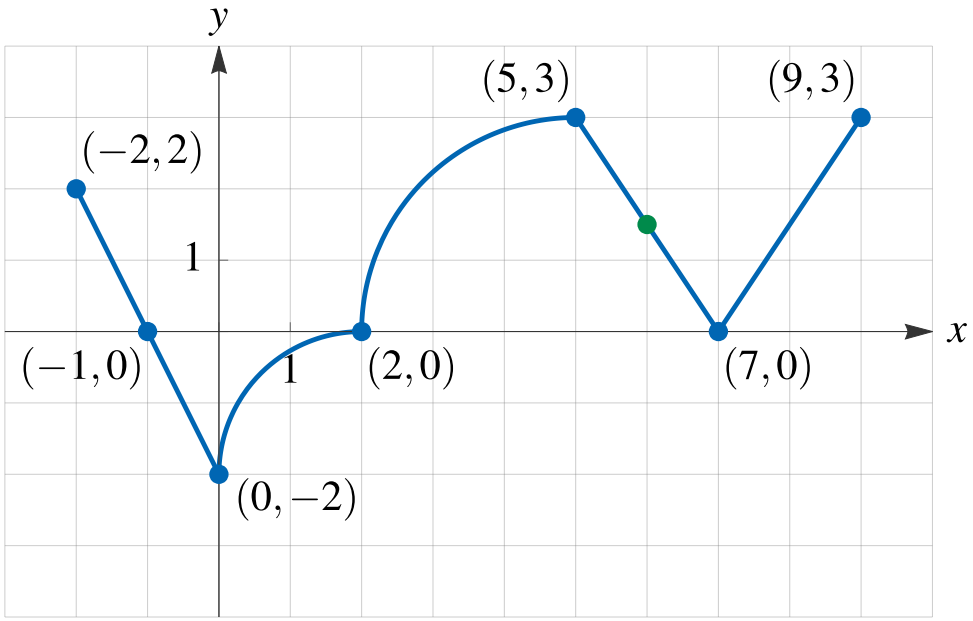
- First two points: earned for considering these x-values as potential locations for the absolute maximum.
- If ordered pairs are presented, read only the x-values for the consideration point.
- Must have $\frac{13}{4}\pi - 4 = 6.21$ identified as the maximum value.
- If an ordered pair is presented, read the value of $f(x)$.
- The table of values is the justification; all values presented must be correct.
- Read with imported values from part (d) and (e).
- It is possible to make an argument using a comparison of areas, but correct values for those areas must be presented.
- A table may include additional x-values; there must be justification involving $g'(x)$ for the maximum value.

(g) Find the value of $g''(6)$, or explain why it does not exist.

Key Concepts

- Geometric interpretation of a derivative: slope of the tangent line to the graph of f at the point where $x = a$.

Solution



$$g''(x) = \frac{d}{dx}[g'(x)] = \frac{d}{dx}[f(x)] = f'(x)$$

$$g''(6) = f'(6) = \frac{3-0}{5-7} = -\frac{3}{2}$$

Scoring Guidelines

(g) $g''(6) = \dfrac{3-0}{5-7} = -\dfrac{3}{2}$	1 : answer

Scoring Notes

- Unlabeled answer is OK.
- If the answer is labeled, it must be correct notation.
- No supporting work necessary.

(h) Must there exist a value of d, for $0 < d < 2$, such that $g'(d)$ is equal to the average rate of change of g over the interval $0 \leq x \leq 2$? Justify your answer.

Key Concepts

Theorem

If f is differentiable at a, then f is continuous at a.

The Mean Value Theorem

Let f be a function that satisfies the following hypotheses:

(1) f is continuous on the closed interval $[a, b]$.

(2) f is differentiable on the open interval (a, b).

Then there is a number c in (a, b) such that

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$

or equivalently,

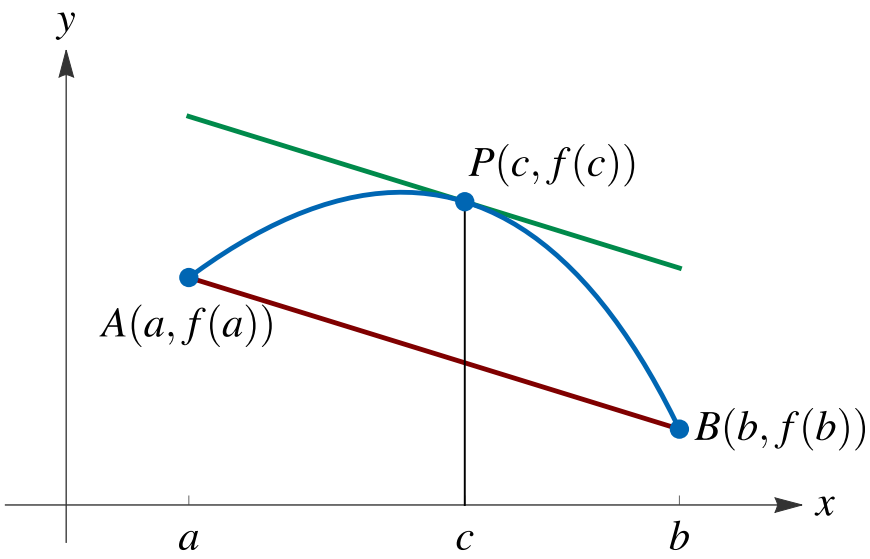
$$f(b) - f(a) = f'(c)(b - a)$$

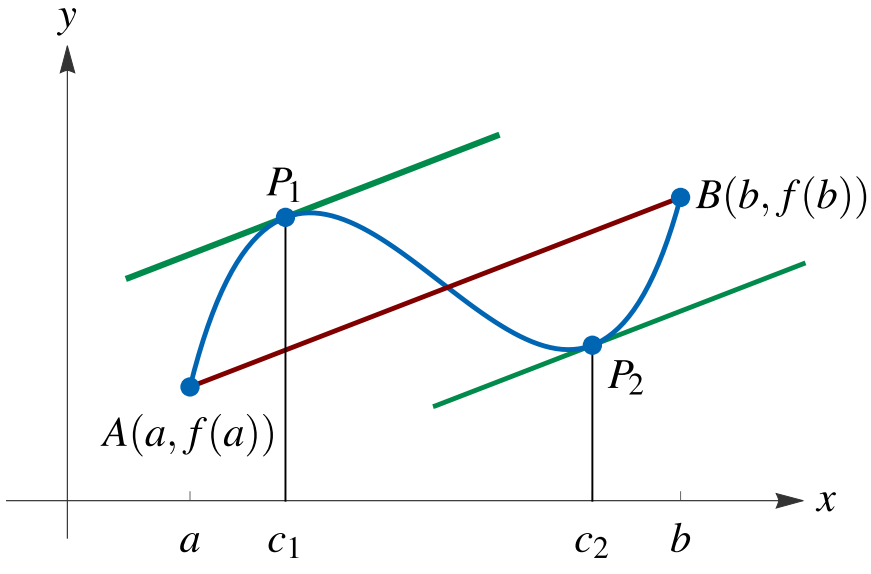
A Closer Look

1. The Mean Value Theorem is an existence theorem.
2. Rolle's Theorem is a special case of the MVT. If $f(a) = f(b)$ then

$$f'(c) = \frac{f(b) - f(a)}{b - a} = \frac{0}{b - a} = 0$$

3. Geometric interpretation:





Solution

$$g(x) = \int_0^x f(t)\,dt \quad \Rightarrow \quad g'(x) = f(x)$$

g is differentiable on $0 < x < 2$

g is continuous on $0 \leq x \leq 2$.

By the MVT, there must exist a value d in $(0, 2)$ such that

$$g'(d) = \frac{g(2) - g(0)}{2 - 0}$$

That is, $g'(d)$ equals the average rate of change of g over the interval $0 \leq x \leq 2$.

Scoring Guidelines

(h) $g' = f \quad \Rightarrow \quad g$ is differentiable on $0 < x < 2 \quad \Rightarrow \quad g$ is continuous on $0 \le x \le 2$

Therefore, the Mean Value Theorem can be applied to g on the interval $0 \le x \le 2$ to guarantee that there exists a value of d, for $0 < d < 2$, such that $g'(d)$ equals the average rate of change of g over the interval $0 \le x \le 2$.

$$2 : \begin{cases} 1 : \text{conditions} \\ 1 : \text{conclusion using Mean Value Theorem} \end{cases}$$

Scoring Notes

- The student must establish that g is a continuous function. The most common, and acceptable, response is *differentiable implies continuous*.
- The student must convey that differentiability implies continuity. Stating g is differentiable *and* continuous, does not earn the first point.
- The response that mentions continuity without establishing continuity is still eligible for the second point.
- The second point requires an answer of *yes* or an equivalent statement.
- Can earn the second point without stating MVT.

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