

TI in Focus: AP[®] Calculus

2018 AP[®] Calculus Exam: AB-1/BC-1
Scoring Guidelines

Stephen Kokoska
Professor, Bloomsburg University
Former AP[®] Calculus Chief Reader

Outline

- (1) Free Response Question
- (2) Scoring Guidelines
- (3) Student performance
- (4) Interpretation
- (5) Common errors
- (6) Specific scoring examples

1. People enter a line for an escalator at a rate modeled by the function r given by

$$r(t) = \begin{cases} 44\left(\frac{t}{100}\right)^3\left(1 - \frac{t}{300}\right)^7 & \text{for } 0 \leq t \leq 300 \\ 0 & \text{for } t > 300, \end{cases}$$

where $r(t)$ is measured in people per second and t is measured in seconds. As people get on the escalator, they exit the line at a constant rate of 0.7 person per second. There are 20 people in line at time $t = 0$.

- (a) How many people enter the line for the escalator during the time interval $0 \leq t \leq 300$?
- (b) During the time interval $0 \leq t \leq 300$, there are always people in line for the escalator. How many people are in line at time $t = 300$?
- (c) For $t > 300$, what is the first time t that there are no people in line for the escalator?
- (d) For $0 \leq t \leq 300$, at what time t is the number of people in line a minimum? To the nearest whole number, find the number of people in line at this time. Justify your answer.

(a) $\int_0^{300} r(t) dt = 270$

According to the model, 270 people enter the line for the escalator during the time interval $0 \leq t \leq 300$.

(b) $20 + \int_0^{300} (r(t) - 0.7) dt = 20 + \int_0^{300} r(t) dt - 0.7 \cdot 300 = 80$

According to the model, 80 people are in line at time $t = 300$.

(c) Based on part (b), the number of people in line at time $t = 300$ is 80.

The first time t that there are no people in line is

$$300 + \frac{80}{0.7} = 414.286 \text{ (or } 414.285) \text{ seconds.}$$

2 : $\begin{cases} 1 : \text{integral} \\ 1 : \text{answer} \end{cases}$

2 : $\begin{cases} 1 : \text{considers rate out} \\ 1 : \text{answer} \end{cases}$

1 : answer

(d) The total number of people in line at time t , $0 \leq t \leq 300$, is modeled by

$$20 + \int_0^t r(x) dx - 0.7t.$$

$$r(t) - 0.7 = 0 \Rightarrow t_1 = 33.013298, t_2 = 166.574719$$

t	People in line for escalator
0	20
t_1	3.803
t_2	158.070
300	80

The number of people in line is a minimum at time $t = 33.013$ seconds, when there are 4 people in line.

$$4 : \begin{cases} 1 : \text{considers } r(t) - 0.7 = 0 \\ 1 : \text{identifies } t = 33.013 \\ 1 : \text{answers} \\ 1 : \text{justification} \end{cases}$$

Student Performance

Part (a)

- Most students earned the integral point: knew the calculus necessary.
- Very few indefinite integrals, few missing differentials: better notational fluency.
- Common errors: used the initial condition, used the rate out.

Part(b)

- Most students knew to use the rate out; presented many ways.
- Some students did not use the initial condition.
- Some students did not consider both rates.

Student Performance

Part(c)

- Students who used their part (a) answer: difficulty arriving at the correct answer.
- Students who used their part (b) answer: more successful, forgot to add 300.

Part (d)

- Many students had difficulty in part (d).
- Many knew they needed to take a derivative and set it equal to 0. However, students had to construct the function.
- Many students consider $r(t) = 0$.
- Justification point was the most difficult for students.

Part (a) 1: integral

$$(1) \int_0^{300} r(t) dt \text{ or } \int_0^{300} 44 \left(\frac{t}{100} \right)^3 \left(1 - \frac{t}{300} \right)^7 dt \quad 1 - ?$$

$$(2) 20 + \int_0^{300} r(t) dt - 0.7t \quad 1 - 0$$

$$(3) 20 + \int_0^{300} r(t) dt - 210 \quad 1 - 0$$

Missing dt : default location is at the end of the expression.

$$(1) \int_0^{300} r(t) \quad 1 - ?$$

$$(2) C + \int_0^{300} r(t) \quad 1 - 0$$

$$(3) \int_0^{300} r(t) + C \quad 0 - 0$$

Part (a) 1: integral

Missing or wrong limits.

$$(1) \int r(t) dt \qquad 0 - ?$$

$$(2) \int r(t) \qquad 0 - ?$$

$$(3) \int_{300}^{300} r(t) dt \qquad 0 - 0$$

Part (a) 1: answer

(1) 270: our answer only.

(2) Bald answer: 0 - 0

Note: If wrong answer in part (a) is > 190 , may import into part (b), or possibly part (c).

Part (b) 1: considers rate out

Earns the point:

$$-0.7t; \int_0^{300} -0.7 dt; -0.7(300); -210; \int_0^t (-0.7) dt$$

The number of people exiting is 210.

Does **not** earn the point:

$$0.7t; \int_0^{300} -0.7t dt; 0.7(300); \int (-0.7) dt; \text{ rate out} = 0.7$$

Missing dt : default location is at the end of the expression.

$$(1) -0.7t + \int_0^{300} r(t) \qquad 1 - ?$$

$$(2) \int_0^{300} r(t) - 0.7t \qquad 0 - 0$$

Part (b) 1: answer

- (1) Earned for our answer: 80.
- (2) Eligibility: earns first point (considers rate out).
- (3) Bald answer: 0 - 0

Examples

- | | |
|---|-------|
| (1) $20 + 270 - 0.7(300)$ | 1 - 1 |
| (2) $20 + A - 0.7(300)$ ($A > 190$, part (a)) | 1 - 1 |
| (3) $290 - 210$ | 1 - 1 |
| (4) $20 + \int_0^{300} r(t) dt = 290 - 0.7(300) = 80$ | 1 - 0 |

Part (c) 1: answer

- (1) 414.286 or 414.285
- (2) Bald answer OK
- (3) If $B > 0$ (wrong) in part (b), may import in part (c).
- (4) Three decimal rule; immunity from further deductions.

Part (d) 1: considers $r(t) - 0.7 = 0$

(1) No need to mention continuity.

(2) $r(t) = 0.7$

1 - ? - ? - ?

(3) In words: Where $r(t) - 0.7$ changes from negative to positive.

(4) Missing dt :

• $n(x) = 20 + \int_0^x r(t) - 0.7x, \quad n' = 0$

? - ? - ? - ?

• $n(x) = 20 + \int_0^x r(t) - 0.7, \quad n' = 0$

1 - ? - ? - ?

(5) Inequalities do not earn this point.

Part (d) 1: identifies $t = 33.013$

• 33.013; 33.01; 33

• $n(x) = 20 + \int_0^x r(t) - 0.7x, \quad n' = 0, \quad t = 33.013$

0 - 1 - ? - ?

Part (d) 1: answers

- (1) Time must be to three decimal places: 33.013
- (2) Number of people must be: 4 or 3.
- (3) 33 0 - 1 - 0 - 0
- (4) 4 at $t = 33.013$ 0 - 1 - 1 - 0

Part (d) 1: justification

- (1) Poorly presented answer (i.e., 33 secs or 3.8 people) can be justified.
- (2) Incorrect answer (i.e., 33.8 secs or -16 people) can not be justified.
- (3) $20 + \int_0^t r(x) dx - 0.7t$: required
- (4) The Candidates Test is justification.
May appear as a list of sentences or phrases.
No value has to be to three decimal places, but presented answers must be.
- (5) First Derivative Test possible: communicate why they are ruling out t_2 .

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