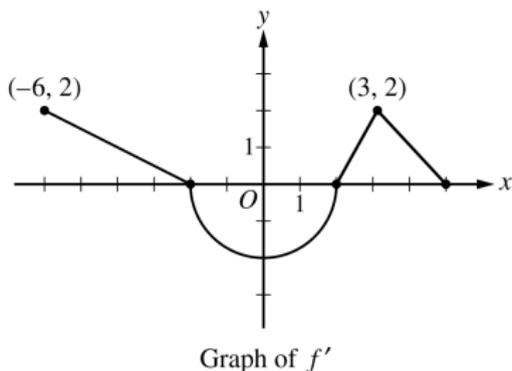


2017 AP Calculus Exam: AB-3/BC-3 Scoring Guidelines

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3. The function f is differentiable on the closed interval $[-6, 5]$ and satisfies $f(-2) = 7$. The graph of f' , the derivative of f , consists of a semicircle and three line segments, as shown in the figure above.
- Find the values of $f(-6)$ and $f(5)$.
 - On what intervals is f increasing? Justify your answer.
 - Find the absolute minimum value of f on the closed interval $[-6, 5]$. Justify your answer.
 - For each of $f''(-5)$ and $f''(3)$, find the value or explain why it does not exist.

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$$(a) f(-6) = f(-2) + \int_{-2}^{-6} f'(x) dx = 7 - \int_{-6}^{-2} f'(x) dx = 7 - 4 = 3$$

$$f(5) = f(-2) + \int_{-2}^5 f'(x) dx = 7 - 2\pi + 3 = 10 - 2\pi$$

(b) $f'(x) > 0$ on the intervals $[-6, -2)$ and $(2, 5)$.

Therefore, f is increasing on the intervals $[-6, -2]$ and $[2, 5]$.

(c) The absolute minimum will occur at a critical point where $f'(x) = 0$ or at an endpoint.

$$f'(x) = 0 \Rightarrow x = -2, x = 2$$

x	$f(x)$
-6	3
-2	7
2	$7 - 2\pi$
5	$10 - 2\pi$

The absolute minimum value is $f(2) = 7 - 2\pi$.

$$3 : \begin{cases} 1 : \text{uses initial condition} \\ 1 : f(-6) \\ 1 : f(5) \end{cases}$$

2 : answer with justification

$$2 : \begin{cases} 1 : \text{considers } x = 2 \\ 1 : \text{answer with justification} \end{cases}$$

AB-3/BC-3

$$(d) f''(-5) = \frac{2-0}{-6-(-2)} = -\frac{1}{2}$$

$$\lim_{x \rightarrow 3^-} \frac{f'(x) - f'(3)}{x - 3} = 2 \quad \text{and} \quad \lim_{x \rightarrow 3^+} \frac{f'(x) - f'(3)}{x - 3} = -1$$

$f''(3)$ does not exist because

$$\lim_{x \rightarrow 3^-} \frac{f'(x) - f'(3)}{x - 3} \neq \lim_{x \rightarrow 3^+} \frac{f'(x) - f'(3)}{x - 3}.$$

$$2: \begin{cases} 1: f'''(-5) \\ 1: f'''(3) \text{ does not exist,} \\ \quad \text{with explanation} \end{cases}$$

AB-3/BC-3

Student Performance

- (1) Most students were able to enter the problem.
- (2) Everywhere a sign: -2 and -6 , and add/subtract area.
- (3) Part (b): communication, open versus closed intervals.
- (4) Issues with local versus global arguments.
- (5) Need to consider the endpoints in the search for global extrema.
- (6) $f''(3)$ difficult. Geometric description versus analytic approach.

AB-3/BC-3

Part (a)

1: uses initial condition

- (1) Earns the point with an appearance in either or both evaluations.
- (2) Philosophy: Uses $f(-2) = 7$ on the way to calculate $f(-6)$ or $f(5)$.
- $f(-6) = 7 + 4$ 1 - 0 - ?
 - $f(-6) = 7 - 4$ 1 - 1 - ?
 - $7 + \int_{-2}^{-6} f'(x) dx$ or $f(-6) = f(-2) + \int_{-2}^{-6} f'(x) dx$ 1 - ? - ?
- (3) The initial condition may appear in the context of an indefinite integral and an attempt to solve for the constant of integration.
- (4) $(-2, 7)$ in a linear approximation or appeal to slope: does not earn first point.
- (5) May appear implicitly.

$$f(5) = 3 + \int_{-6}^5 f'(x) dx \quad 1 - ? - ?$$

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Part (a)

1: $f(-6)$

$$(1) f(-6) = 7 + \int_{-2}^{-6} f'(x) dx = 7 - 4 \qquad 1 - 1 - ?$$

$$(2) f(-6) = 7 - \int_{-6}^{-2} f'(x) dx = 7 - 4 \qquad 1 - 1 - ?$$

$$(3) f(-6) = 7 - 4 = 3 \qquad 1 - 1 - ?$$

1: $f(5)$

$$(1) f(5) = 7 + \int_{-2}^5 f'(x) dx = 7 - 2\pi + 3 \qquad 1 - ? - 1$$

$$(2) f(5) = 7 - 2\pi + 3 = 10 - 2\pi \qquad 1 - ? - 1$$

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Part (a)

$f(-6)$ and $f(5)$

- (1) Read unlabeled work in the correct order.
- (2) Correct and bald: 0 - 1 - 1
- (3) Linkage errors:

$$f(-6) = \int_{-2}^{-6} f'(x) dx = -4 + 7 = 3 \qquad 1 - 0 - 1$$

$$f(5) = \int_{-2}^5 f'(x) dx = -2\pi + 3 + 7 \qquad 1 - 0 - 1$$

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Part (b)

2: answer with justification

- (1) Inclusion or exclusion of endpoints does not matter for scoring.
- (2) "The graph" refers to $f'(x)$. "above the x -axis" is read as "positive."
- (3) Complete correct intervals with $f'(x) > 0$ 2/2
- (4) $x \leq -2$ and $x \geq 2$ because $f'(x)$ is greater than zero for $x < -2$ and $x > 2$. 2/2
- (5) Complete correct intervals without justification or unclear but not false justification attempt. 1/2
- (6) One correct interval alone with $f'(x) > 0$. 1/2
- (7) One correct interval with additional correct subinterval and $f'(x) > 0$. 1/2
- (8) An interval outside the domain, but correct and justified on $[-6, 5]$ 1/2
- (9) Ambiguous: "the function," "the derivative," and "the slope."
- (10) Any interval containing points where $f'(x) < 0$ 0/2

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Part (c)

1: considers $x = 2$

- (1) Earned for considering $x = 2$ as a potential location for the absolute minimum.
- (2) $x = 2$ or $f(2)$ appears in part (c).
- (3) If ordered pairs presented, read only the x -values for the consideration point.

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Part (c)

1: answer with justification

- (1) Must have $f(2) = 7 - 2\pi = 0.7168$ identified as the minimum value
- (2) A minimum presented as a point will be read for the value of $f(x)$.
- (3) Correct evaluations of $f(-6)$, $f(2)$, and $f(5)$,
and $7 - 2\pi$ identified as the minimum. 1 - 1
- (4) May evaluate only $f(-6)$ and $f(2)$, but must include correct reasoning for excluding $f(5)$ as a candidate for the minimum.
- (5) Read imported values for $f(-6)$ and $f(5)$ if greater than $7 - 2\pi$.
- (6) Possible to justify using a comparison of areas; specific and correct.

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Part (d)

1: $f''(-5)$

(1) $f''(-5) = -\frac{1}{2}$ or $-\frac{2}{4}$ or -0.5

(2) Answer may be given without a difference quotient.

(3) Answer does not need to be simplified.

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Part (d)

1: $f''(3)$ does not exist, with explanation

(1) Scoring Guidelines: limit evaluations, statement, not equal.

(2) $f''(3)$ does not exist because $\lim_{x \rightarrow 3^-} f''(x) \neq \lim_{x \rightarrow 3^+} f''(x)$? - 1

(3) $\lim_{x \rightarrow 3^-} \frac{f''(x) - f''(3)}{x - 3} \neq \lim_{x \rightarrow 3^+} \frac{f''(x) - f''(3)}{x - 3}$? - 0

(4) $\lim_{x \rightarrow 3^-} \frac{f(x) - f(3)}{x - 3} \neq \lim_{x \rightarrow 3^+} \frac{f(x) - f(3)}{x - 3}$? - 0

(5) $\lim_{x \rightarrow 3^-} f'(x) \neq \lim_{x \rightarrow 3^+} f'(x)$? - 0

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Part (d)

1: $f''(3)$ does not exist, with explanation

Explanations that earn the second point: (? - 1)

$f''(3)$ does not exist because $f'(x)$ has:

- a corner; a sharp turn; a sharp point; a kink; a cusp;
- an apex of two segments; no distinct tangent line.

Explanations that do **not** earn the second point:

$f''(3)$ does not exist because $f'(x)$ has:

- vertex; absolute value; peak; bounce; node; abrupt change in slope;
- comes to a point; is not differentiable; not smooth; does not have a slope.