Building Concepts: What is a Ratio?

TEACHER NOTES

Lesson Overview

This TI-Nspire™ lesson helps students to understand the concept of ratios. Ratios associate two or more quantities that vary together. Ratios, which are distinct from fractions, are typically noted in the form *a*:*b,* which is the ratio of *a* to *b*. The ratio of two quantities *a* and *b* is written as *a*:*b* or as *b*:*a*.

Equivalent ratios arise by multiplying or dividing each measurement in a ratio pair by the same positive number. Thus, *a*:*b* is equivalent to *c*:*d* if $a = k \cdot c$ and $b = k \cdot d$ where *k* is a positive number

Specific language is used to make the relationship within a ratio explicit: *2 to 3; 2 for every 3; 2 out of every 5; 2 cups to 5 cups*.

Learning Goals

1. Understand ratios as an association of two or more quantities;
2. use ratio language to describe the relationship between two quantities;
3. identify and create equivalent ratios by multiplying each measurement in a ratio pair by the same positive number.

Prerequisite Knowledge

What is a Ratio? is the first lesson in a series of lessons that explore the concepts of ratios and proportional reasoning. Lessons in this series build on the knowledge from previous lessons. Prior to working on this first lesson, students should understand:

- how to compare numbers;
- the concepts of multiplication and division.

Vocabulary

- **ratio:** a pair of values representing two (or more) quantities that vary in the same relative relationship.
- **equivalent ratios:** two ratios such that the values in one ratio can be obtained by multiplying both values in the second ratio by a number greater than zero.

Lesson Pacing

This lesson should take 50–90 minutes to complete with students, though you may choose to extend, as needed.

Lesson Materials

- Compatible TI Technologies:

  TI-Nspire CX Handhelds, TI-Nspire Apps for iPad®, TI-Nspire Software

- What is a Ratio_Student.pdf
- What is a Ratio_Student.doc
- What is a Ratio.tns
- What is a Ratio_Teacher Notes
- To download the TI-Nspire activity (TNS file) and Student Activity sheet, go to http://education.ti.com/go/buildingconcepts.

Class Instruction Key

The following question types are included throughout the lesson to assist you in guiding students in their exploration of the concept:

Class Discussion: Use these questions to help students communicate their understanding of the lesson. Encourage students to refer to the lesson as they explain their reasoning. Have students listen to your instructions. Look for student answers to reflect an understanding of the concept. Listen for opportunities to address understanding or misconceptions in student answers.

Student Activity: Have students break into small groups and work together to find answers to the student activity questions. Observe students as they work and guide them in addressing the learning goals of each lesson. Have students record their answers on their student activity sheet. Once students have finished, have groups discuss and/or present their findings. The student activity sheet can also be completed as a larger group activity, depending on the technology available in the classroom.

Additional Discussion: These questions are provided for additional student practice and to faciliate a deeper understanding and exploration of the content. Encourage students to explain what they are doing and to share their reasoning.

Mathematical Background

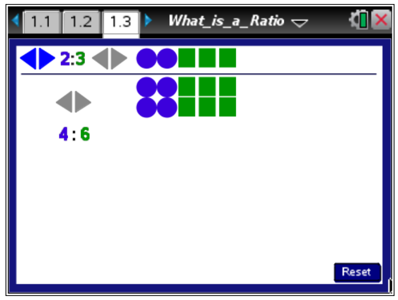
Ratios associate two or more quantities that vary together. As defined by the Common Core State Standards, a quantity involves measurement of an attribute, either discrete (5 oranges) or continuous (5 inches). The measurements may relate to physical attributes such as length, area, volume, distance, or other attributes that can be measured, such as cost. Sometimes the quantities have the same units (e.g., 2 cups of juice and 5 cups of water); other times the quantities differ (e.g., 4 meters and 3 seconds). Ratios can be used for situations in which units are the same and those in which the units are different. The words that are used to indicate a ratio make the relationship explicit: *2 to 3; 2 for every 3; 2 out of every 5; 2 cups to 5 cups*. A ratio is distinct from a fraction although later activities in the *Building Concepts: Ratios and Proportional Reasoning* series illustrate how a ratio can be associated with a fraction, called the value of the ratio. Because ratios are different from fractions, ratio notation should be different from fraction notation, typically a:b is the ratio of a to b. The ratio of two quantities a and b is written as a:b or as b:a.

Equivalent ratios arise by multiplying or dividing each measurement in a ratio pair by the same positive number. Thus, a:b is equivalent to c:d if $a = k \cdot c$ and $b = k \cdot d$ where k is a positive number. Students often confuse the multiplicative aspect of equivalent ratios with addition, i.e., multiplicative change versus additive change. As students engage in activities related to ratios, be sure to highlight the notion of multiplicative change. For example, the ratio 2:3 is equivalent to 10:15, which is 5 times each value in the original ratio pair. Adding 1 to each value in the original ratio pair would produce 3:4, which is not equivalent to 2:3.

Part 1, Page 1.3

Focus: What is a ratio and when are two ratios equivalent?

Page 1.3 displays blue and green arrows at the top. The arrows at the top change the number of circles and squares to set a ratio. The arrows below the line generate equivalent ratios. To reset the page, select **Reset.**



Encourage students to explore how changing the ratio of circles to squares at the top of the screen affects the ratio of circles to squares below the line.

The following questions for Part 1 introduce students to the concept of a ratio as paired quantities, in this case circles and squares, that vary together in the same relationship. This leads to the notion of equivalent ratios. The patterns in the visual representations of equivalent ratios should be associated with the original ratio to reinforce the definition of equivalence.

TI-Nspire Technology Tips

Use the tab key to toggle between the sets of arrows on the screen; note that the colored arrow set is active.

The **Reset** button or ctrl del returns the screen to the original ratio and ratio representation

Arrows on the keypad can also be used in addition to the arrows on the screen to change the values.

Class Discussion

On page 1.3, the circles and squares are said to be in the ratio 2 to 3. The notation for a ratio uses a colon, 2:3.

Have students…	**Look for/Listen for…**
• ***Interpret the ratio. What do you think it means to say two quantities have a 2:3 ratio?***	Answers will vary. Possible answer: There are 2 of one quantity for every 3 of another.
• ***Use the lower arrow several times to change the number of circles and squares. What is happening to the number of circles and squares? To the ratio of circles to squares?***	Answer: The number of squares increases by 3 for every 2 more circles. The ratio increases by multiples of the two numbers in the ratio pair 2: 3.
• ***Describe what you think the picture of circles and squares will be for the ratio 12:18. Use the TNS activity to check your prediction.***	Answer: 12 circles arranged in 2 columns and 6 rows and 18 squares in 3 columns and 6 rows, with each row having 2 circles and 3 squares.

Class Discussion (continued)

• **Reset** ***the page. Use the lower arrow several times, observing how the pictures and ratios change. How is the way the circles and squares are arranged in the display related to the ratio?***	Answer: Each one has 2 columns of circles and 3 columns of squares. The first time I use the lower arrow it adds a row in each column, doubling the number of circles and squares for a ratio of 4:6. The second time I use the arrow it adds another row, multiplying the circles and squares by 3.
• ***For the ratio 2:3, will you ever have 12 circles and 21 squares? Explain your thinking.***	Answer: No, the changes in the numbers 2:3 are not the same for both of the values. You need the same number of rows for both circles and squares, and for 12 circles you would have 2 columns with 6 rows and for 21 squares you would have 3 columns with 7 rows. So, 12:21 will never happen.

Student Activity Questions—Activity 1

1. **For each of the following, describe the shapes in the initially stated ratio. Then predict how the numbers of shapes will change in the new ratios. Finally, check your answers using the TNS activity.**

   a. **The ratio is 1 to 5.**

   Answer: There will be 1 circle to 5 squares, and as you increase the ratios, it will change to 2 circles and 10 squares, 3 circles and 15 squares, and so on.

   b. **The ratio is 6 to 1.**

   Answer: There will be 6 circles and 1 square, and as you increase the ratios, there will be 12 circles and 2 squares, 18 circles and 3 squares, and so on.

   c. **The ratio is 1:1.**

   Answer: There will be 1 circle and 1 square, and as you increase the ratio, the number of circles and squares will be equal.

2. ***Equivalent ratios*** **are ratios formed by multiplying or dividing each quantity in a given ratio by a common positive number. The arrows above the line on page 1.3 change the ratio. Suppose you set the ratio 4 circles to 3 squares.**

   a. **Use the lower arrow to create a new ratio; record the ratios you see. Are these ratios equivalent? Why or why not?**

   Answer: The ratios are equivalent because the values in each ratio pair are multiples of 4:3.

   b. **Is 8:7 equivalent to 4:3? Why or why not?**

   Answer: The ratios are not equivalent because both of the values in the ratio have to be multiplied by the same number. 8 is twice 4, but twice 3 gives 6 not 7.

Student Activity Questions—Activity 1 (continued)

3. Leave the original ratio at 4:3. Use the TNS activity to help you answer each question. Then explain how you could answer the question without the TNS activity.

a. If you have 16 circles, how many squares will you have?

Answer: 12. You could have figured it out because the values in a ratio equivalent to 4:3 will be the same number multiplying both 4 and 3. 16 is 4×4 so you multiply 3 by 4 to get 16:12.

b. If you have 18 squares, how many circles will you have?

Answer: 24. 6×3 is 18, so you need 6×4 to get 24.

c. If the total number of circles and squares is 35, how many of each will you have?

Answer: 20 circles and 15 squares. You can find the answer by making a list of the equivalent ratios and finding the sum of the two quantities. Or, since there are 7 objects in each row, there must be 5 rows. [4:3 is equivalent to $5 \times 4:3 = (20:15)$]

Class Discussion

Encourage students to use the TNS activity to further examine and identify ratios that are equivalent to a given ratio.

Suppose you had a ratio of 3 circles to 5 squares.

- ***List at least three different equivalent ratios that would show up when you selected the lower arrow.***
  Answers will vary. Possible answers: 6:10; 9:15; 12:20
- ***Set the ratio to 3:5 and check your answer.***

Have students…	Look for/Listen for…
Use the TNS activity to help you find the ratio in each case if it is possible to do so. The number of circles is	
• ***always twice the number of squares?***	Answer: Yes, the ratio is 2:1 or any equivalent ratio.
• ***always one fourth of the number of squares?***	Answer: Yes, the ratio is 1:4 or any equivalent ratio.
• ***always one less than the number of squares?***	Answer: No, the ratio could be 1:2, but an equivalent ratio could be 2:4 and the first value in the ratio pair is not 1 less than the second value.

Class Discussion (continued)

Guide students in examining the relationship between each term in a ratio and the corresponding terms in equivalent ratios.

If you were to set the ratio to be 5:7, will you ever get

- ***45 circles and 63 squares? Why or why not?***

  Answer: Yes, 45:63 is $9 \times 5{:}9 \times 7$, so it is a multiple of 5:7.

- ***25 circles and 75 squares? Why or why not?***

  Answer: No, 25:75 is not a multiple of 5:7.

- ***Lori says that the values in equivalent ratios are always a multiple of a common ratio. Tomas says that you can get equivalent ratios by adding. Use examples from the TNS activity to explain to Lori and Tomas whose reasoning is correct and why*.**

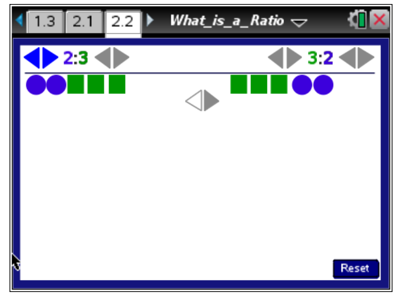
  Answer: Lori is correct. For the ratio 4:7, any equivalent ratio will have a common multiplier like 3 to get 12:21. This keeps the "balance" of 4 circles to 7 squares. You can group them into sets of 4 circles and 7 squares.

  If you added 2 to both you would have 6:9 and now the relationship between the circles and squares is different with 2 circles for every 3 squares, which is not the same as 4 circles for every 7 squares.

  Tomas could be correct if he explained that by adding he meant that you add 4 to the first component of the ratio and 7 to the second component of the ratio repeatedly: 12:21 + 4:7---16:28 = $4 \times$ (4:7). Tomas might also be correct if he meant you could add multiples of 4 and 7 to the original ratio 4:7 to get (12 + 4):(21 + 7) or 16:28, which is 4 times the original ratio and so equivalent.

Part 2, Page 2.2

Focus: Expressing ratios as *a*:*b* or *b*:*a*.

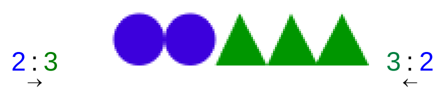


Questions are intended to show that typically ratios have no inherent order, *a*:*b* could be expressed as *b*:*a*. Have students discuss the similarities and differences between the groups of shapes that illustrate the ratios 2:3 and 3:2. Have them use the arrows on the pages to generate other pairs of ratios that show reverse order.

Class Discussion

Guide students in a discussion about the order of the terms in a ratio.

Have students…	Look for/Listen for…
• ***Explain how 3:2 and 2:3 shown on page 2.2 are related.***	Answer: They both could describe the situation with 3 circles and 2 squares.
• ***Do you think this will always be the case? Why or why not?***	Answer: 3:2 will be the same as 2:3 only when the quantities attached to 3 and 2 are not changed. In general, 2:3 could be 2 circles and 3 squares while 3:2 could be 3 squares and 2 circles. It is important to know the units for each quantity.



Student Activity Questions—Activity 2

1. For any ratio equivalent to 2:3, is the number of circles divided by 2 the same as the number of squares divided by 3? Explain why or why not.

Answer: Any ratio equivalent to 2:3 will be of the form 2c:3c for some positive c. So if you divide the first by 2 and the second by 3, you will always end up with the same number, c.

2. Suppose the ratio was 5 to 3. If there were a total of 120 circles and squares, how many squares would there be? Explain how you found your answer.

Answers will vary. Possible answer: One way is to write them all out: 12 × 5:12 × 3 for a total of 96, then 13 × 5:13 × 3 for a total of 65 + 39 = 104; 14 × 5:14 × 3 for a total of 70 + 42 112; 15 × 5:15 × 3 for a total of 75 + 45 = 120, which is the answer. There are 8 objects in each row, so there would be 120/8 = 15 rows. [$15\times(5:3)=75:45$) = 75:45]

Sample Assessment Items

After completing the lesson, students should be able to answer the following types of questions. If students understand the concepts involved in the lesson, they should be able to answer the following questions without using the TNS activity.

1. Which of the following pairs of ratios are equivalent?

   a. 6:3 and 5:10 b. 6:3 and 3:6 c. 6:3 and 12:6 d. 3:1½ and 6:3

   Answer: c and d

2. Write two ratios that describe the relationship between the number of students and number of pencils.

   Answers might be 3:4 and 18:24.

3. If you need 9 books for every 4 students, how many books will you need for 12 students?

   Answer: 27

4. Which of the following will produce an equivalent ratio to 7:5?

   a. adding 2 to both values

   b. multiplying both values by 2

   c. dividing the first value by 7 and the second value by 5

   d. multiplying the first value by 5 and the second value by 7

   Answer: b

Student Activity Solutions

In these activities you will work together to explore ratios and equivalent ratios. After completing each activity, discuss and/or present your findings to the rest of the class.

Activity 1 [Page 1.3]

1. For each of the following, describe the shapes in the initially stated ratio. Then predict how the numbers of shapes will change in the new ratios. Finally, check your answers using the TNS activity.

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   Answer: There will be 1 circle to 5 squares, and as you increase the ratios, it will change to 2 circles and 10 squares, 3 circles and 15 squares, and so on.

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   c. The ratio is 1:1.

   Answer: There will be 1 circle and 1 square, and as you increase the ratio, the number of circles and squares will be equal.

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   Suppose you set the ratio 4 circles to 3 squares.

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   c. If the total number of circles and squares is 35, how many of each will you have?

   Answer: 20 circles and 15 squares. You can find the answer by making a list of the equivalent ratios and finding the sum of the two quantities. Or, since there are 7 objects in each row, there must be 5 rows. [4:3 is equivalent to $5 \times 4:3 = (20:15)$]

Activity 2 [Page 2.2]

1. For any ratio equivalent to 2:3, is the number of circles divided by 2 the same as the number of squares divided by 3? Explain why or why not.

   Answer: Any ratio equivalent to 2:3 will be of the form 2c:3c for some positive c. So if you divide the first by 2 and the second by 3, you will always end up with the same number, c.

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   Answers will vary.
   Possible answer: One way is to write them all out: $12 \times 5 : 12 \times 3$ for a total of 96, then $13 \times 5 : 13 \times 3$ for a total of 65 + 39 = 104; $14 \times 5 : 14 \times 3$ for a total of 70 + 42 112; $15 \times 5 : 15 \times 3$ for a total of 75 + 45 = 120, which is the answer. There are 8 objects in each row, so there would be 120/8 = 15 rows. [$15 \times (5 : 3) = 75 : 45$) = 75:45]