Building Concepts: Ratios Within and Between Scaled Shapes

TEACHER NOTES

Lesson Overview

In this TI-Nspire™ lesson, students learn that ratios are connected to geometry in multiple ways. When one figure is an enlarged or reduced copy of another by some scale factor, the ratios of corresponding lengths between the figures are proportional in accordance with that scale factor. Students should recognize that lengths scale by a linear scale factor, but areas scale by the square of the scale factor that relates the lengths (if the area is measured in the unit of measurement derived from that used for length).

To find unknown lengths from known lengths, students can set up proportions in tables or equations. Students can also reason multiplicatively.

Learning Goals

1. Solve problems involving proportional relationships in geometric figures when one figure is scaled up or down from the other;
2. recognize that ratios of lengths within an object are preserved in figures that are enlarged or reduced;
3. identify the relationship between a scale factor and the corresponding sides of two figures when one is an enlarged or reduced copy of another;
4. recognize that if a figure is enlarged or reduced by a scale factor, all lengths associated with the figure (i.e. height, perimeter) scale by the same scale factor;
5. recognize that area scales by the square of the linear scale factor of the sides of a figure.

Prerequisite Knowledge

Ratios Within and Between Scaled Shapes is the final lesson in a series of lessons that investigates ratios and proportional relationships. In this lesson, students explore the connection between ratios and geometry. This lesson builds on the knowledge presented in *Proportional Relationships, Solving Proportions,* and *Ratios and Scaling*. Prior to working on this lesson, students should:

- understand how to find the perimeter and area of a rectangle.

Vocabulary

- **scale factor:** the ratio formed by the lengths of the corresponding sides of two figures when one is a scaled copy of the other. This ratio might be expressed as the value of the ratio, the fractional value associated with the ratio.

Lesson Pacing

This lesson should take 50–90 minutes to complete with students, though you may choose to extend, as needed.

Lesson Materials

- Compatible TI Technologies:

  TI-Nspire CX Handhelds, TI-Nspire Apps for iPad®, TI-Nspire Software
- Ratios Within and Between Scaled Shapes_Student.pdf
- Ratios Within and Between Scaled Shapes_Student.doc
- Ratios Within and Between Scaled Shapes.tns
- Ratios Within and Between Scaled Shapes_Teacher Notes
- To download the TI-Nspire lesson (TNS file) and Student Activity sheet, go to http://education.ti.com/go/buildingconcepts.

Class Instruction Key

The following question types are included throughout the lesson to assist you in guiding students in their exploration of the concept:

Class Discussion: Use these questions to help students communicate their understanding of the lesson. Encourage students to refer to the TNS lesson as they explain their reasoning. Have students listen to your instructions. Look for student answers to reflect an understanding of the concept. Listen for opportunities to address understanding or misconceptions in student answers.

Student Activity: Have students break into small groups and work together to find answers to the student activity questions. Observe students as they work and guide them in addressing the learning goals of each lesson. Have students record their answers on their student activity sheet. Once students have finished, have groups discuss and/or present their findings. The student activity sheet can also be completed as a larger group activity, depending on the technology available in the classroom.

Additional Discussion: These questions are provided for additional student practice, and to faciliate a deeper understanding and exploration of the content. Encourage students to explain what they are doing and to share their reasoning.

Mathematical Background

In this TI-Nspire™ lesson, students learn that ratios are connected to geometry in multiple ways. When one figure is an enlarged or reduced copy of another by some scale factor, the ratios of corresponding lengths between the figures are proportional in accordance with that scale factor. The internal ratio of two sides in one figure is equivalent to the ratio of the corresponding two sides in the second figure. To find unknown lengths from known lengths, students can set up proportions in tables or equations. Students can also reason multiplicatively by 1) applying a scale factor that relates lengths in two different figures, or 2) considering the ratio of two lengths within one figure, finding a multiplicative relationship between those lengths, and applying that relationship to the ratio of the corresponding lengths in the other figure. (*Ratio and Proportional Relationship Progressions*, 2011). In the first case, students are looking at a relationship across figures (external proportions), and in the second they are using a relationship within a figure (internal proportions).

Students should recognize that lengths scale by a linear scale factor, but areas scale by the square of the scale factor that relates the lengths (if the area is measured in the unit of measurement derived from that used for length). That is, if the sides of a figure are changed by a scale factor of x, the area will be changed by a scale factor of x^2.

The lesson can be repurposed and directly tied to similarity although the questions use *enlarged and reduced copies of a figure* rather than using the term *similar*.

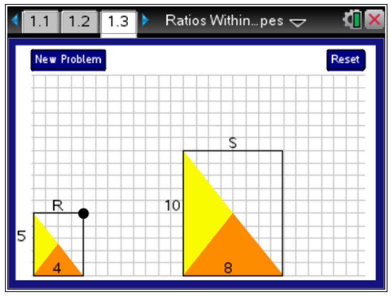
Part 1, Page 1.3

Focus: What relationships that can be described by ratios exist within and between two shapes when one is a scaled copy of the other?

On page 1.3, one rectangle is an enlarged or reduced copy of the other.

Change the size of the shape by dragging the black dot at one of the vertices on the rectangle or using the arrow keys.

Change the shapes displayed by selecting **New Problem** on the screen or using the menu key to select **New Problem.**



TI-Nspire Technology Tips

Use the tab key to toggle between the black dot

Reset returns to the original screen or press ctrl del on handheld to reset.

Teacher Tip: Point out that the yellow triangles measure the lengths of the rectangles and that the orange triangles measure the widths. Have students write the widths and the lengths of the triangles as ratios. Then have them calculate the scale factor.

Class Discussion

These questions review the notions of corresponding sides between two scaled shapes and the concept of scale factor.

Look at the two rectangles on page 1.3.

- ***Which side of the larger rectangle "corresponds" to the side of length 5 in the smaller rectangle?***

  Answer: the side of length 10

- ***What is the ratio of the sides between the bases of the rectangles? the heights of the rectangles?***

  Answer: In both cases the ratio is 2:1 (or 1:2).

Have students…	Look for/Listen for…
Look at the two rectangles on page 1.3.	
• ***What is the ratio of the perimeters of the two figures? Justify your answer.***	Answer: The perimeter of the smaller rectangle is 18 and the perimeter of the larger square is 36, so the ratio is 18:36 or 1:2.

Class Discussion (continued)

Have students…	Look for/Listen for…
• ***Make a conjecture about what you think will be the ratio between the diagonals. Explain your thinking.***	Answer: Students might think the same ratio holds. If they know about the Pythagorean theorem, they can actually prove that the scale factor is the same: the diagonal of the smaller rectangle is $\sqrt{(25+16)}=\sqrt{41}$ and of the larger rectangle is $\sqrt{(100+64)}=\sqrt{164}=2\sqrt{41}$. The ratio of the diagonals will be $\sqrt{41}:2\sqrt{41}$, which is 1:2.
Drag the black dot so the base of the smaller rectangle is 5 and the height is 8.	
• ***What is the ratio of the base to the height of the smaller rectangle? of the larger rectangle?***	Answer: 5:8 for both rectangles
• ***What is the ratio of the corresponding sides in the two rectangles?***	Answer: 1:2
Select New Problem.	
• ***How will your answers to the two previous change? Justify your reasoning***.	Answers will vary depending on which rectangle is generated in this problem: For example, if the rectangle in question 2a has a ratio of base to height that is 5:8, and the rectangle generated in this problem has a ratio of base to height as 7.5:12, then the ratios of the rectangles in the two problems are the same. Both 7.5:12 and 5:8 are equivalent because they are both equivalent to 60:96 (multiply both values in 5:8 by 12 and both values in 7.5:12 by 8). For question 2b, the ratios between the corresponding sides will be 2:3 because 8:12 is equivalent to 2:3 (divide both values by 4) and 5:7.5 is equivalent to 2:3 (divide both values by 2.5).

- ***If all the side lengths of a figure are multiplied by the same positive number, the number is called a scale factor, and the new figure is a scaled copy of the original figure. What is the scale factor between the two rectangles? Explain what this means in terms of the side lengths of the rectangles.***

Answers will vary: Each side of the smaller rectangle is $\frac{2}{3}$ the side of the larger rectangle or you could say that each side of the larger rectangle is $\frac{3}{2}$ ($1\frac{1}{2}$) as large as a side of the smaller rectangle.

Class Discussion (continued)

- ***Create two rectangles so that each side of one rectangle is $2\frac{1}{2}$ times as large as the sides of the other rectangle. Explain why your answer is reasonable.***

Answers will vary: For example, the smaller rectangle could have a base of 5 and a height of 4 and the larger one could have a base of 12.5 and a height of 10 because $2\frac{1}{2}$ times 5 is 12.5 and $2\frac{1}{2}$ times 4 is 10. In another example, the base of one rectangle could be 4 and the height 6 and the other rectangle could have a base of 10 and a height of 15, which is reasonable because $2\frac{1}{2}$ times 4 is 10 and $2\frac{1}{2}$ times 6 is 15.

> **Teacher Tip:** You may want students to explain that $2\frac{1}{2}$ times 5 is 2 fives and $\frac{1}{2}$ of another 5 or $10+2.5=12.5$.

- ***What is the scale factor between your two rectangles?***

Answer: $\frac{5}{2}$ or $2\frac{1}{2}$

Use the TNS activity. Describe two rectangles such that a scale factor will produce Rectangle S so that it is:

- ***a reduction of Rectangle R.***

Examples may vary, but the only possible scale factor is $\frac{1}{2}$ given the constraints of the TNS lesson.

- ***an enlarged copy of Rectangle R.***

Examples will vary; any scale factor other than $\frac{1}{2}$ (on the page) will work.

- ***Use a scale factor of 3 to produce Rectangle S from Rectangle R. Would you say Rectangle S is 3 times as large as Rectangle R? Why or why not?***

  Answer: “As large” is very vague because it does not clarify whether it refers to area or perimeter. The measures of the sides of S are three times the measures of the sides of R; however the area of S is nine times larger than the area of R.

- ***With the same scale factor as in part c could you say that length of the base of Rectangle R is $\frac{1}{3}$ the length of the base of Rectangle S? Why or why not?***

  Answer: Yes, because if the length of the base in S is 3 times the length of the base in R (which is what a scale factor means), then dividing by 3 will give $\frac{1}{3}$ of the base in S.

Class Discussion (continued)

These questions allow students begin to investigate ratios between measures related to two scaled shapes other than the lengths of the sides, including the ratio of the areas.

Reset the page. Move the dot on Rectangle R to have the base 4 and the height 6.

- ***What is the scale factor?***

  Answer: 2

- ***What is the ratio of the perimeters of Rectangles R and S?***

  Answer: 2

- ***Use the grid to find the height of the orange triangle in Rectangle R if the base is 4, and the height of the orange triangle in Rectangle S if the base is 8. How are the heights of the two triangles related?***

  Answer: heights are 3 and 6; the height of the triangle in *S* is twice the height of the triangle in *R*.

- ***Find the height of the yellow triangle in Rectangle R if the base is 4 and the height of the yellow triangle in the Rectangle S if the base is 8. How are these two measures related?***

  Answer: heights are 2 and 4; one is twice the other.

Use the answers you found for the group of questions above to help answer the following questions.

- ***Find the areas of the yellow and orange triangles in Rectangle R.***

  Answer: They are both 6 square units.

- **Is your answer to the question above surprising? Why or why not?**

  Possible answer: The answer is surprising because the triangles are not the same shape.

- ***Find the area of the yellow triangles in each rectangle. How are the areas related?***

  Answer: The area of the yellow triangle in Rectangle *R* is 6 square units, and the area of the yellow triangle in Rectangle *S* is 24 square units; the area of the triangle in Rectangle *S* is 4 times the area of the triangle in Rectangle *R*.

- ***Can you find the area of the orange triangle in Rectangle S without using the grid? Explain why or why not***.

  Possible answers: The areas of the orange and yellow triangles in each rectangle are the same; so knowing that the area of the yellow triangle in *S* is 24 square units means the area of the orange triangle in *S* is also 24 square units.

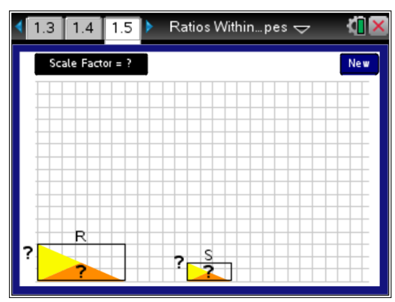
Additional Discussion

On page 1.3, choose New Problem until you find a way to create a Rectangle R with base 4 and height 5 and have the given scale factor for Rectangle S. Find the ratios of the corresponding sides of the figure on the right to the figure on the left for the given scale factors.

Scale Factor	Internal: Ratio of height to base in Rectangle S	Internal: Ratio of height to base in Rectangle R	External: Ratios of base of Rectangle S to base of Rectangle R	External: Ratios of height of Rectangle S to height of Rectangle R
$\frac{1}{2}$	[2.5:2]	[5:4]	[1:2]	[1:2]
1	[5:4]	[5:4]	[1:1]	[1:1]
$\frac{3}{2}$	[7.5:6]	[5:4]	[3:2]	[3:2]
2	[10:8]	[5:4]	[2:1]	[2:1]
$\frac{5}{2}$	[12.5:10]	[5:4]	[5:2]	[5:2]

Part 2, Pages 1.5 and 1.7

Focus: The questions in this part engage students in thinking about the minimum amount of information they need to determine the measures of all of the sides of two scaled rectangles**.** The TNS lesson is designed to be self-checking, so students can experiment with different combinations and make and test conjectures.



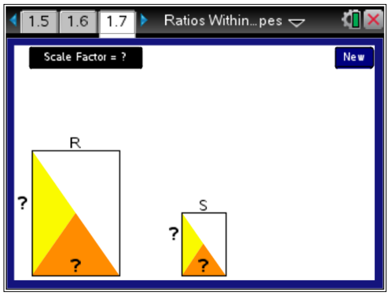
Once students are comfortable with scale factors and moving between the shapes, Part 2, which involves files that provide solutions to verify student conjectures, could be done as the second part of this lesson where students work with the files and share ideas with each other.

On page 1.5, you can choose up to five values by:

- selecting them from the sides of the shapes, selecting **Scale Factor**, or
- using the tab key to toggle between selections and pressing enter key to choose.

Select a new problem by pressing **New Problem** or by using the menu key to select the option.

Page 1.7 behaves like page 1.5, but does not have a grid.



Teacher Tip: Note that the scale factor and side lengths are randomly generated so students may have different responses. Sharing their examples and thinking could help them begin to generalize about scale factors as a way to move from one figure to another and connect corresponding parts of scaled figures. Examining the internal ratios among the sides of each shape is another way to make a connection between the shapes.

Student Activity Questions—Activity 1

1. **On page 1.5, Rectangle S is a scaled copy of Rectangle *R*. Reveal the scale factor and the height of Rectangle *R*.**

   a. **Do you know anything about the dimensions of Rectangle *S*? Explain your thinking.**

   Answer: The height of Rectangle *S* will be the product of the scale factor and the height of Rectangle R because one rectangle is just scaled up or down from the other by the scale factor. Note that the answers to the question should be the same, but the heights and scale factors may be different as they are randomly generated (for example, with a scale factor of 0.5, the height of Rectangle *R* is 7, corresponding height in Rectangle *S* will be $\frac{1}{2}$ of 7 or 3.5; with a scale factor of 1.5, the height of Rectangle *R* is 11, height of Rectangle *S* will be is (1.5)(11) or 16.5.

   b. **Reveal the height of Rectangle *S* to check your thinking.**

   c. **Now reveal the base of Rectangle *S*. What is the base of Rectangle *R*?**

   Answers will vary depending on the randomly generated dimensions: For example, if the base of *S* is 2, then using the scale factor, the base of *R* will be twice that or 4.

2. **Select New**.

   a. **Select enough information to enable you to find the ratio of the corresponding sides in the two rectangles.**

   Answers will vary: Students might select just the scale factor, and they would be correct. Others might select corresponding heights in each rectangle or the corresponding bases.

Student Activity Questions—Activity 1 (continued)

b. **What is the ratio of the perimeters of the two rectangles? Explain how you found your answer.**

   Answers will vary: Students might know that the scale factor of the perimeters is the same as the scale factor of the corresponding sides. You should ask the students to defend this reasoning. Others might want to find the sides, compute the perimeters, and then find the ratio.

c. **What is the ratio of the areas of the two rectangles? Explain your reasoning.**

   Answer: Find the lengths of the missing sides; the product of the base and height for each rectangle will give the area of each rectangle, which can be used to find the ratio of the areas.

d. **How do the ratio of the corresponding sides, the ratio of the perimeters, and the ratio of the areas compare? Use the TNS lesson to generate several examples to help think about the problem. Use one of the examples to explain your reasoning.**

   Answers will vary depending on the example: The scale factor will give you the ratio of the corresponding sides and the ratio of the perimeter. The square of the scale factor will give the ratio of the areas.

3. **Page 1.7 has five missing pieces of information: a scale factor and the height and base of two different rectangles. Answer each of the following questions and use an example to explain your reasoning.**

   a. **Select to find the measures of any three of the lengths. Predict the scale factor and what the missing length will be.**

      Possible answer: I selected the height of R and got 8 and the height and base of S and got 4 and 4, respectively. I predicted the scale factor would be $\frac{1}{2}$ because the two rectangles are scaled copies of each other and if S is a square, R should be one also, and so the base of R would be twice as big as the base of S or 8.

   b. **Select New. Select the bases of the two rectangles. Does this give you information about any of the other 3 missing pieces?**

      Possible answer: The base of R was 2 and the base of S was 4, so the scale factor is 2, but I cannot find any of the other pieces of information.

   c. **Select New. Select the height of one rectangle, the base of the other. Does this give you information about the other three missing pieces of information?**

      Possible answer: No, if the height of R is 11 and the base of S is 2, I need some other information to connect the two rectangles.

4. **Select New. Select what you think you will need to be able to figure out the other pieces of information. Share what you have done with a partner and have them find the missing information for your problem. Use the TNS lesson to check their answer.**

   Answers will vary.

Additional Discussion

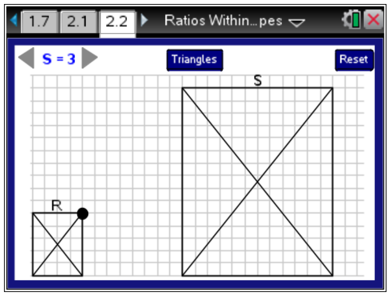
Have students…	Look for/Listen for…
Explain whether it is possible to determine all five values with just two of the five values	Answer: No, it is not possible with just two values. In order to determine the scale factor you need a pair of corresponding sides (one from each rectangle). To find all of the dimensions, you need the length of one more side in either rectangle. If you have the scale factor, you need the base from one rectangle and the height from the other in order to find all the dimensions of the rectangles.
Why does the lesson not allow you to select four of the five question marks?	Answer: Because you do not need four pieces of information to find all of the pieces. If you knew the scale factor, two sides of one rectangle and one of the other, you have extra information because the scale factor can be found from looking at corresponding sides in the two rectangles.
Describe the exact combinations of the fewest number of the five pieces of information you need to determine the other pieces.	Answer: You can use 1. scale factor, base of R, height of S 2. scale factor, height of R, base of S 3. height and base of R, height of S 4. height and base of R, base of S 5. height and base of S, height of R 6. height and base of S, base of R

Building Concepts: Ratios Within and Between Scaled Shapes

TEACHER NOTES

Part 3, Page 2.2

Focus: Connecting the relationship of the scale factor to the ratio of areas of scaled shapes.



On page 2.2, the arrows on the top left set a scale factor. Change the size of the shape by dragging or using the arrow keys to move the black dot at the top right of Rectangle R.

Change the scale factor using the arrows at the top of the page or by using the arrow keys on the keypad.

Show the solid triangles within the shape by selecting the **Triangles** button. When the button is highlighted you can press enter to show the triangles.

Use the tab key to toggle between the Shape, Scale Factor, and Triangles.

Teacher Tip: Students can use the grid to determine measures as well as the visual representation of how a scale factor of x, which is a linear measure, produces x^2 copies of the area in a scaled figure. The files display many different patterns that can be investigated by students who are interested. A challenge might be to have students create a set of questions for their classmates to explore.

Class Discussion

Drag the dot on Rectangle R to create a rectangle with a base of 6 and a height of 2. Which of the following statements do you think are true? Explain your reasoning.

- ***In Rectangle R, triangles with base 6 have larger area than the triangles with base 2.***

  Answer: This not true because the areas of the two triangles are both 3 square units.

- ***In Rectangle S, the triangle with base 6 has a smaller area than the triangle with base 18.***

  Answer: False because both triangles have area 27 square units.

- **Select Triangles. The area of the purple triangle in Rectangle R is 6 square units.**

  Answer: True because the area will be $6 \times 2 \times \frac{1}{2}$.

- ***Every triangle in Rectangle S has an area that is 3 times the area of the corresponding triangle in Rectangle R.***

  Answer: False because the areas are 9 times the area of the triangles in Rectangle R.

- ***The orange and yellow triangles in Rectangle S have the same area.***

  Answer: True because the areas are both 27 square units.

Student Activity Questions—Activity 2

1. **Select the yellow triangle in Rectangle S.**

   a. **Describe what changed.**

   Answer: The yellow triangle is tiled or covered with 9 smaller triangles.

   b. **How do you think the yellow triangle in Rectangle R is related to the yellow triangles in the large yellow triangle in Rectangle S? Explain your reasoning.**

   Answer: The small yellow triangles used to tile the triangle in Rectangle S seem to be exact copies of the yellow triangle in Rectangle R. You can tell from the grid that the first column of five triangles in Rectangle S all have bases of 5 and heights of 2, so they each have an area of 5, the same area with the same base and height as the yellow triangle in Rectangle R. The other four yellow triangles in Rectangle S seem to be just flipped copies of the ones in the first column. So there are 9 triangles, each with an area 5 square units for a total area of 45 square units.

   c. **Make a prediction about the number of orange triangles from Rectangle R that will tile the orange triangle in Rectangle S. Select the orange triangle in S to check your prediction**.

   Answer: 9 triangles

2. **Change the scale factor to 2.**

   a. **Predict how many purple triangles in Rectangle R will tile the purple triangle in Rectangle *S*.**

   Answer: 4

   b. **What is the area of the orange triangle in Rectangle *S*? Explain your reasoning.**

   Answer: 20 square units, because the area of the orange triangle in Rectangle *R* is 5 and 4(5) = 20.

3. **Given the scale factor of 4, which of the following describes the relationship among the areas of the yellow triangle in Rectangle R and the yellow triangle in Rectangle S? Explain how the scale factor helps you answer the question and why it does.**

   a. **The area of the triangle in S is four times the area of the triangle in R.**

   b. **The area of the triangle in S is $\frac{1}{4}$ times the area of the triangle in R.**

   c. **The area of the triangle in S is 16 times the area of the triangle in R.**

   d. **The area of the triangle in S is $\frac{1}{16}$ the area of the triangle in R.**

   Answer: The area of Rectangle R will be scaled by 16 because each dimension is scaled by 4 and $4 \times 4 = 16$. So choice c is correct; the area of the yellow triangle in Rectangle S will be 16 times the area of the yellow triangle in Rectangle R.

Student Activity Questions—Activity 2 (continued)

4. Tim made the following claims. Use an example from the TNS lesson and explain what you would say to Tim.

a. The internal ratios of two sides of a figure will stay the same in a reduced or enlarged copy of the figure.

b. The ratio of the perimeters of a figure and a scaled copy will be the same as the ratio associated with the scale factor.

c. The ratio of the areas of a figure and a scaled copy will be the same as the ratio associated with the scale factor.

d. If the scale factor is $\frac{3}{4}$ each length in a scaled copy of a figure will be $\frac{3}{4}$ of the length in the original figure.

Answer: All but c are true. Examples will vary.

Additional Discussion

Have students…	Look for/Listen for…
• ***Reset the page. Select Triangles. Describe the ratios you see in the figures, using the measurements for your rectangles.***	Answers will vary. Possible ratios are the internal ratios, base to height (or height to base) in each of the figures, the scale factor that determines the ratio of the corresponding heights or bases in the two rectangles. For example, a base of 4, a height of 5 in Rectangle R; a base of 12, a height of 15 in the rectangle would give internal ratios of 4 to 5 (or 5 to 4); base to base or height to height would give the ratio 1 to 3 (or 3 to 1), which is the scale factor. Some may even describe the ratios of the perimeters and/or area, diagonals, etc.
Describe Rectangles R and S if:	
• ***the scale factor is 4, and the area of the yellow triangle on Rectangle S is 48 square units.***	Possible answer: Any dimensions for R that will produce an area for the yellow triangle in Rectangle R of 3 square units, i.e., a base of 4 and a height of 3 (or base 3 and height 4)
• ***the height of the orange triangle in Rectangle R is 2, the area of the yellow triangle in Rectangle S is 48, square units.***	Possible answer: With a scale factor of 4, the base of Rectangle R could be 3, so R is 3×4 and Rectangle S is 9×12.

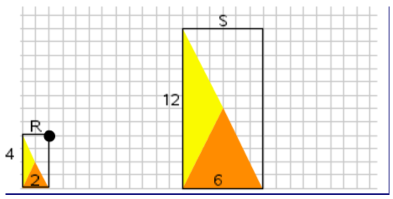
Additional Discussion (continued)

Have students…	Look for/Listen for…
• ***the area of the purple triangle in Rectangle R is 15 square units and the scale factor is 2.***	Possible answer: Rectangle R could be 5×6 and Rectangle S would be 10×12.
Which of the following statements are true if Figure ABCD is a scaled copy of Figure RSTV? Explain your thinking. (You may want to draw the figure and use examples from the lesson to help your thinking.)	
• ***The ratio of AB to BC will identify the scale factor.***	Answer: False. This will give the internal ratio, not the scale factor.
• ***The ratio of BC to ST is the same as the ratio of CD to TV.***	Answer: True because BC and ST are corresponding sides of the two rectangles and so are CD and TV, and they will be related by the scale factor.
• ***The ratio of AB to BC is the same as the ratio of RS to ST.***	Answer: True because they will have the same internal ratio because one figure is a copy of the other.
• ***The ratio of AB to TV is the same as the ratio of DC to ST.***	Answer: False because the sides are not corresponding; AB and DC are opposite sides in the rectangle, while TV and ST are adjacent sides.

Sample Assessment Items

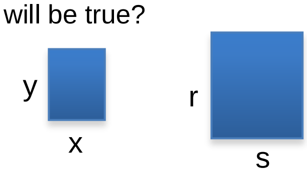
After completing the lesson, students should be able to answer the following types of questions. If students understand the concepts involved in the lesson, they should be able to answer the following questions without using the TNS lesson.

1. If the Rectangle S is an enlarged copy of Rectangle R, what is the scale factor?



Answer: 3

2. The larger rectangle is a scaled up copy of the smaller rectangle. Which of the following statements will be true?



a. $\frac{y}{x} = \frac{s}{r}$

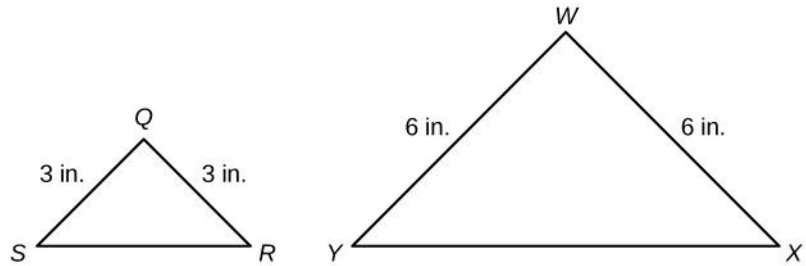
b. $\frac{y}{r} = \frac{x}{s}$

c. $\frac{y}{s} = \frac{r}{x}$

d. $\frac{y}{x} = \frac{r}{s}$

Answer: b) $\boldsymbol{\frac{y}{r} = \frac{x}{s}}$ ***,d)*** $\boldsymbol{\frac{y}{x} = \frac{r}{s}}$

3. The triangle on the left is a reduced copy of the triangle on the right.



Which statement must be true?

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a. *YX* is 6 inches long.

b. *SR* is twice as long as *YX*.

c. The ratio of *XY* to *RS* is 2 to 1.

d. The ratio of *QS* to *SR* is 1 to 1.

Answer: c) The ratio of XY to RS is 2 to 1.

4. One right triangle is an enlarged copy of the other. If the following statements are true, draw and label the two triangles. Find the missing measures.

a. The ratio of *AB* to *RS* is $\frac{2}{3}$.

b. *CB* is 6 cm long.

c. TR is 15 cm long.

Answer: The triangles are ABC and RST. For Triangle ABC, $AB = 8$, $AC = 10$, ***and*** $BC = 6$. ***For Triangle RST,*** $RS = 12$, $TS = 9$, ***and*** $TR = 15$.

Building Concepts: Ratios Within and Between Scaled Shapes

TEACHER NOTES

Student Activity Solutions

In these activities you will solve problems involving proportional relationships in geometric figures when one figure is scaled up or down from the other. After completing the activities, discuss and/or present your findings to the rest of the class.

Activity 1 [Pages 1.5; 1.7]

1. On page 1.5, Rectangle S is a scaled copy of Rectangle R. Reveal the scale factor and the height of Rectangle R.

   a. Do you know anything about the dimensions of Rectangle S? Explain your thinking.

   Answer: The height of Rectangle S will be the product of the scale factor and the height of Rectangle R because one rectangle is just scaled up or down from the other by the scale factor. Note that the answers to the question should be the same, but the heights and scale factors may be different as they are randomly generated (for example, with a scale factor of 0.5, the height of Rectangle R is 7, corresponding height in Rectangle S will be $\frac{1}{2}$ *of 7 or 3.5; with a scale factor of 1.5, the height of Rectangle R is 11, height of Rectangle S will be is (1.5)(11) or 16.5.*

   b. Reveal the height of Rectangle S to check your thinking.

   c. Now reveal the base of Rectangle S. What is the base of Rectangle R?

   Answers will vary depending on the randomly generated dimensions: For example, if the base of S is 2, then using the scale factor, the base of R will be twice that or 4.

2. Select **New**.

   a. Select enough information to enable you to find the ratio of the corresponding sides in the two rectangles.

   Answers will vary: Students might select just the scale factor, and they would be correct. Others might select corresponding heights in each rectangle or the corresponding bases.

   b. What is the ratio of the perimeters of the two rectangles? Explain how you found your answer.

   Answers will vary: Students might know that the scale factor of the perimeters is the same as the scale factor of the corresponding sides. You should ask the students to defend this reasoning. Others might want to find the sides, compute the perimeters, and then find the ratio.

   c. What is the ratio of the areas of the two rectangles? Explain your reasoning.

   Answer: Find the lengths of the missing sides; the product of the base and height for each rectangle will give the area of each rectangle, which can be used to find the ratio of the areas.

d. How do the ratio of the corresponding sides, the ratio of the perimeters, and the ratio of the areas compare? Use the TNS lesson to generate several examples to help think about the problem. Use one of the examples to explain your reasoning.

Answers will vary depending on the example: The scale factor will give you the ratio of the corresponding sides and the ratio of the perimeter. The square of the scale factor will give the ratio of the areas.

3. Page 1.7 has five missing pieces of information: a scale factor and the height and base of two different rectangles. Answer each of the following questions and use an example to explain your reasoning.

a. Select to find the measures of any three of the lengths. Predict the scale factor and what the missing length will be.

Possible answer: I selected the height of R and got 8 and the height and base of S and got 4 and 4, respectively. I predicted the scale factor would be $\frac{1}{2}$ *because the two rectangles are scaled copies of each other and if S is a square, R should be one also, and so the base of R would be twice as big as the base of S or 8.*

b. Select **New**. Select the bases of the two rectangles. Does this give you information about any of the other 3 missing pieces?

Possible answer: The base of R was 2 and the base of S was 4, so the scale factor is 2, but I cannot find any of the other pieces of information.

c. Select **New**. Select the height of one rectangle, the base of the other. Does this give you information about the other three missing pieces of information?

Possible answer: No, if the height of R is 11 and the base of S is 2, I need some other information to connect the two rectangles.

4. Select **New.** Select what you think you will need to be able to figure out the other pieces of information. Share what you have done with a partner and have them find the missing information for your problem. Use the TNS lesson to check their answer.

Answers will vary.

Activity 2 [Page 2.2]

1. Select the yellow triangle in Rectangle S.

a. Describe what changed.

Answer: The yellow triangle is tiled or covered with 9 smaller triangles.

b. How do you think the yellow triangle in Rectangle R is related to the yellow triangles in the large yellow triangle in Rectangle S? Explain your reasoning.

Answer: The small yellow triangles used to tile the triangle in Rectangle S seem to be exact copies of the yellow triangle in Rectangle R. You can tell from the grid that the first column of five triangles in Rectangle S all have bases of 5 and heights of 2, so they each have an area of 5, the same area with the same base and height as the yellow triangle in Rectangle R. The other four yellow triangles in Rectangle S seem to be just flipped copies of the ones in the first column. So there are 9 triangles, each with an area of 5 square units for a total area of 45 square units.

c. Make a prediction about the number of orange triangles from Rectangle R that will tile the orange triangle in Rectangle S. Select the orange triangle in S to check your prediction.

Answer: 9 triangles

2. Change the scale factor to 2.

a. Predict how many purple triangles in Rectangle R will tile the purple triangle in Rectangle S.

Answer: 4

b. What is the area of the orange triangle in Rectangle S? Explain your reasoning.

Answer: 20 square units, because the area of the orange triangle in Rectangle R is 5 and 4(5) = 20.

3. Given the scale factor of 4, which of the following describes the relationship among the areas of the yellow triangle in Rectangle R and the yellow triangle in Rectangle S? Explain how the scale factor helps you answer the question and why it does.

a. The area of the triangle in S is four times the area of the triangle in R.

b. The area of the triangle in S is $\frac{1}{4}$ times the area of the triangle in R.

c. The area of the triangle in S is 16 times the area of the triangle in R.

d. The area of the triangle in S is $\frac{1}{16}$ the area of the triangle in R.

Answer: The area of Rectangle R will be scaled by 16 because each dimension is scaled by 4 and $4 \times 4 = 16$ *. So choice c is correct; the area of the yellow triangle in Rectangle S will be 16 times the area of the yellow triangle in Rectangle R.*

4. Tim made the following claims. Use an example from the TNS lesson and explain what you would say to Tim.

   a. The internal ratios of two sides of a figure will stay the same in a reduced or enlarged copy of the figure.

   b. The ratio of the perimeters of a figure and a scaled copy will be the same as the ratio associated with the scale factor.

   c. The ratio of the areas of a figure and a scaled copy will be the same as the ratio associated with the scale factor.

   d. If the scale factor is $\frac{3}{4}$ each length in a scaled copy of a figure will be $\frac{3}{4}$ of the length in the original figure.

   Answer: All but c are true. Examples will vary.