Building Concepts: Introduction to Rates

TEACHER NOTES

Lesson Overview

This TI-Nspire™ lesson helps students to understand the concept of rates. Every ratio has an associated unit rate. In this lesson, students will explore the relationship between ratios, rates, and unit rates. They investigate unit rates by determining how to give out bags of dog food in order for each dog to have the same amount.

> Unit rates can be used to compare amounts in different ratios.

Learning Goals

1. Identify the unit rate associated with a ratio;
2. recognize that unit rates appear with 1 as the second value in the ratio pair, a:1;
3. interpret language associated with unit rates;
4. recognize that equivalent ratios have the same unit rate.

Prerequisite Knowledge

Introduction to Rates is the second lesson in a series of lessons that explore the concepts of ratios and proportion. Prior to working on this lesson, students should:

- understand equivalent fractions, unit fractions, and multiplication and division of fractions;
- have completed *What is a Ratio?*

Vocabulary

- **rate:** a ratio that compares two unlike quantities.
- **unit rate:** a ratio of the form a, to one unit: $\frac{a}{1}$.

🕐 Lesson Pacing

This lesson should take 50–90 minutes to complete with students, though you may choose to extend, as needed.

Lesson Materials

- Compatible TI Technologies:

  TI-Nspire CX Handhelds, TI-Nspire Apps for iPad®, TI-Nspire Software

- Introduction to Rates_Student.pdf
- Introduction to Rates_Student.doc
- Introduction to Rates.tns
- Introduction to Rates_Teacher Notes
- To download the TI-Nspire activity (TNS file) and Student Activity sheet, go to http://education.ti.com/go/buildingconcepts.

Class Instruction Key

The following question types are included throughout the lesson to assist you in guiding students in their exploration of the concept:

Class Discussion: Use these questions to help students communicate their understanding of the lesson. Encourage students to refer to the TNS activity as they explain their reasoning. Have students listen to your instructions. Look for student answers to reflect an understanding of the concept. Listen for opportunities to address understanding or misconceptions in student answers.

Student Activity: Have students break into small groups and work together to find answers to the student activity questions. Observe students as they work and guide them in addressing the learning goals of each lesson. Have students record their answers on their student activity sheet. Once students have finished, have groups discuss and/or present their findings. The student activity sheet can also be completed as a larger group activity, depending on the technology available in the classroom.

Additional Discussion: These questions are provided for additional student practice, and to faciliate a deeper understanding and exploration of the content. Encourage students to explain what they are doing and to share their reasoning.

Mathematical Background

Every ratio has an associated unit rate. For example, the ratio 5 feet for every 2 seconds has the associated rate $\frac{5}{2}$ feet for every 1 second; the ratio 3 cups water for every 2 cups orange concentrate has the associated unit rate $\frac{3}{2}$ cups water for every 1 cup orange concentrate. In Grades 6 and 7, students describe rates in terms such as "for each one," "for every," "for each", and "per." The *unit rate* is the numerical part of the rate; the "unit" in "unit rate" is the 1 in "1 per student," "for each 1 minute," or "for every 1 cup." Unit rates are important in understanding slope as a rate of change and as a problem-solving strategy for finding solutions to problems involving proportional relationships.

Part 1, Page 1.3

Focus: What is a unit rate?

This part focuses on developing the concept of a unit rate and how to use it to compare amounts in given different ratios. Students should pay attention to the words used to describe a rate such as *per* and *each*. The questions provide students with practice using operations with fractions.
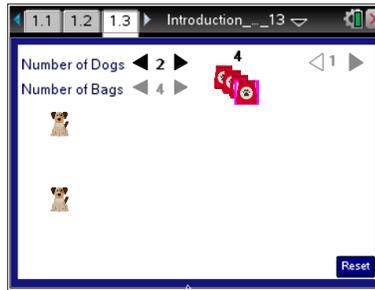
On page 1.3, the horizontal arrows on the left will set the number of dogs and a number of bags of dog food to be shared between the dogs. Each bag of dog food can be dragged to a position next to a dog.

The arrows at the upper right of the page will divide the bags into equal parts.

To move the food to the dogs:

- Drag the bags into place, or
- Use the up/down arrows on the handheld to move the bags of food into position. Use enter to set a bag down by a dog.

TI-Nspire Technology Tips

Press the tab key to toggle between each set of arrows

The **Reset** button or ctrl del returns the screen to the original ratio and ratio representation

Arrows on the keypad can also be used in addition to the arrows on the screen.

📌 Class Discussion

Teacher Tip: Be sure students relate the ratio of dogs to bags of dog food. Relate the numbers in the ratio to the pictures on the screen. Drag whole bags of food to each dog, then work with students to explore how to partition one remaining bag of dog food into fractions of a bag so that it can be distributed among the dogs. Have students explore different ways to share the bags of dog food when they have fewer bags of dog food than number of dogs.

If there are 2 dogs and 4 bags of dog food and the dog food is shared equally among the dogs, how many bags of dog food are there for each dog? Move the bags of dog food on page 1.3 to answer the question.

Answer: Each dog gets 2 bags of dog food.

Have students…	Look for/Listen for…
Reset page 1.3, then change the number of dogs to 3 and answer the questions below.	
• *How is this problem different from the question 1 where there were 2 dogs where there were 2 dogs?*	Answer: The number of bags of dog food is not a multiple of 3. Each dog gets 1 bag of dog food, and then there is only 1 bag left to divide among the 3 dogs.
• *Use the fraction arrow to partition the bags of dog food. What can you do to have the same amount of dog food for each dog?*	Answer: Give each dog one bag of dog food. Then divide the remaining bag into thirds. Each dog gets $1\frac{1}{3}$ bags of dog food.
• *Change the number of bags to 3. How much dog food will each of the 3 dogs get?*	Answer: Each dog gets 1 bag of dog food.

👥 Student Activity Questions

1. **Use the TNS activity to answer the following: Suppose you have 4 dogs. How many bags of dog food would you need if you wanted:**

   a. $\frac{1}{2}$ **bag per dog?**

   Answer: 2 bags of dog food

   b. $2\frac{1}{2}$ **bags per dog?**

   Answer: 10 bags of dog food

   c. $1\frac{1}{4}$ **bags per dog?**

   Answer: 5 bags of dog food

2. **If you have 2 of one quantity for 1 of another quantity, the ratio is 2:1, and the unit rate is 2. Which of the following specifies a unit rate?**

   a. **3 bags of dog food: 1 dog**

   b. **1 bag of dog food per 3 dogs**

   c. **Each dog has 4 bags of dog food.**

   Answer: They could all be thought of as unit rates: 3 bags per 1 dog; 3 dogs per 1 bag; 4 bags per 1 dog.

Student Activity Questions (continued)

3. **The amount of standing room is important for lines and crowds. Which of the following will give a unit rate for the number of feet per person given the ratio 5 feet for every 10 people? Explain your reasoning.**

   a. **Divide both numbers by 5.**

   b. **Divide both numbers by 10.**

   c. **Multiply both numbers by 2.**

   Answer: The correct answer is (b) because it gives you $\frac{1}{2}$ of a foot for every person. (a) gives you 1:2 or 1 foot for every 2 people or 2 people per foot, but that is not what the question asked. (c) gives you 10 to 20 which is not a unit rate.

   > **Teacher Tip:** The following question provides an opportunity for students to revisit equivalent ratios. Remind students that to find equivalent ratios, they must multiply or divide *both* terms of the ratio by the same positive number.

4. **Identify the rates that are equivalent to $\frac{24}{18}$ to 1. Explain your thinking.**

   a. $\frac{12}{9}$ **to 1**   b. $\frac{4}{3}$ **to 1**   c. $\frac{20}{15}$ **to 1**   d. $\frac{28}{22}$ **to 1**

   Answer: a, b, and c are equivalent because the numerator and denominator of $\frac{24}{18}$ can both be divided by 2 to get a) $\frac{12}{9}$; by 6 to get b) $\frac{4}{3}$; by dividing by 6 then multiplying by 5 (or multiplying by $\frac{5}{6}$) to get c) $\frac{20}{15}$. You cannot multiply or divide 24 and 18 by anything to get 28 and 22.

5. **Which of the following strategies would you use to find the number of bags of dog food if you have 6 dogs and 44 bags of dog food for each dog? Explain your thinking. (You can check your thinking by using the TNS activity.)**

   a. **Tom says to make marks by each of the dogs, counting the total number of marks up to 44, until you don't have enough to make another mark by every dog. Then figure out what fraction of the leftover bags would give all 6 dogs the same fraction of a bag.**

   b. **Terri said she would figure out how close a multiple of 6 gets to 44 and then figure out how to divide the remaining bags into fractions to give every dog the same amount.**

   c. **Tess said she would figure out how many times she can subtract 6 from 44 and see what is left. Then she would divide the leftover bags into fractional parts so everyone would have the same fraction of a bag.**

d. Tim said he would look at the ratio 44:6 and divide both values in the ratio by 6 to get $\frac{44}{6}:1$.

e. Taura said she would give every dog lots–maybe 5 bags each would use up 30 bags. Then, she would give the other bags out 1 by 1 until she had less than 6 bags left. She would divide the leftover bags into fractional parts so each dog would have the same fractional amount of dog food.

Answer: They are all correct and would give $7\frac{1}{3}$ bags of dog food per dog. Student thinking will vary depending on which one they chose.

6. If you went 246 miles on 9 gallons of gas, which of the correct strategies above would be easiest to use to find the number of miles you traveled? Explain why.

Possible answers: $\frac{246}{9}$ to 1 miles per gallon; $\frac{82}{3}$ to 1 miles per gallon; $27\frac{1}{3}$ miles per gallon.

Students might recognize that Tim's strategy would be easy to use in this problem.

📌 **Class Discussion**

- *Write a unit rate for the number of bags per dog for the following. Use the TNS activity to support your reasoning.*
  - *10 bags for 4 dogs*

    Answer: $\frac{5}{2}$ bags per dog or $2\frac{1}{2}$ bags per dog

  - *7 bags for 3 dogs*

    Answer: $\frac{7}{3}$ bags per dog or $2\frac{1}{3}$ bags per dog

  - *9 bags for 6 dogs*

    Answer: $\frac{3}{2}$ bags per dog or $1\frac{1}{2}$ bags per dog

- *Use unit rates to find which of the two situations would give the most dog food per dog. Explain your thinking in each case.*
  - *10 bags for every 5 dogs or 6 dogs for every 12 bags*

    Answer: 10 bags: 5 dogs would be 2 bags per dog; 6 dogs: 12 bags would be 1 dog per 2 bags. The dogs would get the same amount in either case.

  - *8 bags for every 6 dogs or 10 bags for every 8 dogs*

    Answer: If you divide both values in the first ratio by 6, 8:6 would be $\frac{4}{3}:1$ or $1\frac{1}{3}$ bags per dog.

    Dividing the values in the second ratio by 2, 10:8 would be $\frac{5}{4}:1$ or $1\frac{1}{4}$ bags per dog. $1\frac{1}{3}$ is

    larger than $1\frac{1}{4}$ so 8 bags for every 6 dogs would give the most per dog.

⊕ Additional Discussion

Have students…	Look for/Listen for…

Explain your reasoning in each case.

- *Sam stated that the unit rates for equivalent ratios will always be equivalent. Do you agree or disagree?*

  Answer: Yes because equivalent ratios are like 6:4 and 9:6. The unit rate for each will be equivalent to $\frac{3}{2}$:1 because when you divide the first value by the second value in the ratio, it will always be $\frac{3}{2}$.

- *Are any of the following equivalent ratios? Explain your reasoning.*

  $1:\frac{1}{3}$ $\frac{1}{3}:1$ $3:1$ $1:3$

  Answer: $1:\frac{1}{3}$ is equivalent to 3:1 because you multiply both quantities in $1:\frac{1}{3}$ by 3 and get 3:1.

  $\frac{1}{3}:1$ is equivalent to 1:3 because you multiply both quantities in $\frac{1}{3}:1$ by 3 to get 1:3.

Use unit rates to answer each of the following. You may want to use the TNS activity to help you think about the problem or draw a picture showing how the dog food would be shared. Explain your reasoning in each case.

- *If you need to have 8 bags of dog food for 4 dogs, how many do you need for 6 dogs?*

  Answer: 8:4 is 2:1, which means 2 bags of dog food for every dog. So, for 6 dogs you would need 12 bags of dog food.

- *If you have $\frac{1}{2}$ bag of dog food on hand for 2 dogs, how many bags would you need for 10 dogs?*

  Answer: $\frac{1}{2}$:2 would be $\frac{1}{4}$:1 or $\frac{1}{4}$ bag per dog. $10 \times \frac{1}{4} = \frac{5}{2}$ so you would need $2\frac{1}{2}$ bags for 10 dogs.

- *A general rule for a pet boarding center is to have on hand 3 bags of dog food for every 4 dogs they board for long-term care. How many bags of dog food should they have if they expect to board 6 dogs?*

  Answer: 3:4 is $\frac{3}{4}$ bag per dog. So for 6 dogs, they would need $6 \times \frac{3}{4}$ or $\frac{18}{4}$ or $4\frac{1}{2}$ bags of dog food.

Sample Assessment Items

After completing the lesson, students should be able to answer the following types of questions. If students understand the concepts involved in the lesson, they should be able to answer the following questions without using the TNS activity.

1. A recipe calls for 3 cups of flour to 4 cups of sugar. How much flour is there per cup of sugar?

    Answer: $\frac{3}{4}$ cup of flour per cup of sugar

2. It took 7 hours to mow 4 lawns, each with about the same area.

    a. What is the rate at which lawns were being mowed?

    Answer: $\frac{7}{4}$ hours to mow one lawn or $1\frac{3}{4}$ hours per lawn.

    b. How many lawns could be mowed in 35 hours?

    Answer: 20 lawns

3. Helen rode 4 km in the first 10 minutes of a race. If she continues at the same pace, how far will she ride in the next 8 minutes?

    Answer: 4:10 is $\frac{2}{5}$ of a km per minute, so in 8 minutes she will ride $8 \times \frac{2}{5}$ or $\frac{16}{5} = 3\frac{1}{5}$ km.

4. One racer drives 26 miles in 4 hours; a second racer drives 34 miles in 6 hours. Which racer drives more miles per hour?

    Answer: 26:4 is the same as $\frac{13}{2}$:1 or $6\frac{1}{2}$ miles per hour. 34:6 is the same as $\frac{34}{6}$:1 = $\frac{17}{3}$:1 or $5\frac{2}{3}$ miles per hour. The first racer drives more miles per hour.

Student Activity Solutions

In this activity you will work together to use unit rates to solve problems. After completing this activity, discuss and/or present your findings to the rest of the class.

Activity [Page 1.3]

1. Use the TNS activity to answer the following: Suppose you have 4 dogs. How many bags of dog food would you need if you wanted

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    c. Multiply both numbers by 2.

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    b. Terri said she would figure out how close a multiple of 6 gets to 44 and then figure out how to divide the remaining bags into fractions to give every dog the same amount.

    c. Tess said she would figure out how many times she can subtract 6 from 44 and see what is left. Then she would divide the leftover bags into fractional parts so everyone would have the same fraction of a bag.

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