Building Concepts: Units Other Than Unit Squares

TEACHER NOTES

Lesson Overview

This TI-Nspire™ lesson helps students to understand the importance of the unit by approaching fractions using area models. A fraction makes sense only if it refers to a particular scale or unit. One half of a unit square can represent different areas depending on the size of the unit. If two unit squares do not have the same scale, a unit fraction will not have the same size area

If two unit squares do not have the same scale, a unit fraction will not have the same size area.

Learning Goals

Students should understand and be able to explain each of the following:

1. Fractions can only be compared if they refer to the same whole;
2. Fractions can only be added or subtracted if they refer to the same whole;
3. It is possible to have a fraction based on one unit square represent a larger area than the same fraction based on a different size unit square.

Prerequisite Knowledge

Units Other Than a Unit Square is the fourteenth lesson in a series of lessons that explore fractions. This lesson can be worked on any time after students have had experience with the lessons *Equivalent Fractions, Fractions and Unit Squares,* and *Comparing Units*. If you chose to use the questions referring to addition and subtraction, students should be familiar with *Adding and Subtracting Fractions with Like Denominators* and *Adding Fractions with Unlike Denominators*. Prior to working on this lesson students should understand:

- the concept of equivalent fractions.
- how to compare fractions.
- how to add and subtract fractions.

Vocabulary

- **equivalent fractions:** fractions that are located at the same point on the number line
- **unit fraction:** a fraction where the numerator is 1

Lesson Pacing

This lesson contains multiple parts and can take 50–90 minutes to complete with students, though you may choose to extend, as needed.

Lesson Materials

- Compatible TI Technologies:

  TI-Nspire CX Handhelds, TI-Nspire Apps for iPad®, TI-Nspire Software

- Units Other Than Unit Squares_Student.pdf
- Units Other Than Unit Squares_Student.doc
- Units Other Than Unit Squares.tns
- Units Other Than Unit Squares_Teacher Notes
- To download the TI-Nspire activity (TNS file) and Student Activity sheet, go to http://education.ti.com/go/buildingconcepts.

Class Instruction Key

The following question types are included throughout the lesson to assist you in guiding students in their exploration of the concept:

Class Discussion: Use these questions to help students communicate their understanding of the lesson. Encourage students to refer to the TNS activity as they explain their reasoning. Have students listen to your instructions. Look for student answers to reflect an understanding of the concept. Listen for opportunities to address understanding or misconceptions in student answers.

✓ **Student Activity Sheet**: The questions that have a check-mark also appear on the Student Activity Sheet. Have students record their answers on their student activity sheet as you go through the lesson as a class exercise. The student activity sheet is optional and may also be completed in smaller student groups, depending on the technology available in the classroom. A (.doc) version of the Teacher Notes has been provided and can be used to further customize the Student Activity sheet by choosing additional and/or different questions for students.

Bulls-eye Question: Questions marked with the bulls-eye icon indicate key questions a student should be able to answer by the conclusion of the activity. These questions are included in the Teacher Notes and the Student Activity Sheet. The bulls-eye question on the Student Activity sheet is a variation of the discussion question included in the Teacher Notes.

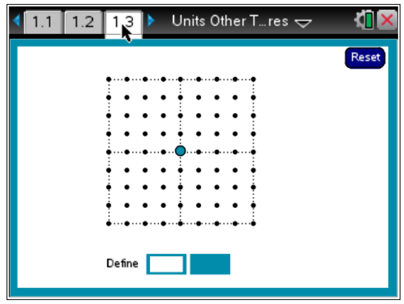
Mathematical Background

This TI-Nspire™ lesson helps students to understand the importance of the unit by approaching fractions using area models. A fraction makes sense only if it refers to a particular scale or unit. One half of a unit square can represent different areas depending on the size of the unit. If two unit squares do not have the same scale, a unit fraction will not have the same size area (i.e., if one unit square uses 1 inch by 1 inch to define the unit and the second uses 1 foot by 1 foot to define the unit, then $\frac{1}{12}$ does not represent the same area for both). A unit does not have to be "regular" in the sense that it could be the shape obtained by a configuration of three small squares or it could be a package of ten energy bars. The question might be 'What fraction is represented by four small squares?' or 'What is $\frac{1}{2}$ of the squares?' If the unit is a package of 10 energy bars, then one half would be 5 bars.

Within this framework, students can be asked to revisit equivalent fractions, comparing fractions, and adding and subtracting fractions, noting that these operations only make sense if the fractions represent the same whole. The lesson might take several days to complete, particularly if students share their strategies and responses, discussing the advantages and disadvantages of them.

Part 1, Page 1.3

Focus: Students will tile a rectangle to solve fraction problems.



On page 1.3, the circle can be dragged to partition the larger rectangle into smaller rectangles, essentially tiling the rectangle. To **Define** a region, select the empty bordered region (the rectangle outline at the bottom of the page) and then on one or more of the rectangles in the figure. This will enclose the selected rectangles in a border. Select a region that has been bordered to delete the border.

TI-Nspire Technology Tips

Students may find it easier to use the tab key to toggle between objects and then use the arrow keys to move or change their selections.

To reset the page, select **Reset** in the upper right corner.

To **Shade** a region, select the shaded rectangle at the bottom of the page and then on rectangle that has a border. This will define a subset of the smaller rectangles in the bordered region. Selecting inside a rectangle in the bordered region will shade that rectangle; selecting a shaded region a second time will delete the shading.

Teacher Tip: Be sure students understand how the interaction with the tiles supports the mathematics. Asking them how the tiling is connected to their thinking about what a “unit” is and what is important about knowing the unit in dealing with fractions can lead to a productive discussion about the mathematical concepts.

Class Discussion

Have students…	Look for/Listen for…
Move the circle to create a tiling of the large rectangle that has 16 pieces.	
• ***Create a region that has six pieces and shade $\frac{2}{6}$ of your region. Use your picture to make the case that $\frac{2}{6}$ is equivalent to $\frac{1}{3}$.***	Answer: The six shaded pieces can be paired into 3 pairs, and you have shaded 1 of the 3 pairs. So $\frac{2}{6}=\frac{1}{3}$

Class Discussion (continued)

- ***Using the same tiling, create a region that has twelve pieces and shade $\frac{4}{12}$. Use your picture to make the case that $\frac{4}{12}$ is equivalent to $\frac{1}{3}$.***

  Answer: One of 3 rows can be shaded so $\frac{4}{12}$ is the same as $\frac{1}{3}$.

- ***Is the $\frac{1}{3}$ in the first problem the same as the $\frac{1}{3}$ in the second problem above? Why or why not?***

  No, the two are different. In the first one, the whole was 6 small rectangles; in the second problem the whole is made of 12 small rectangles. So in the first, $\frac{1}{3}$ was 2 of the six, but in the second $\frac{1}{3}$ was 4 of the 12, which was a larger area.

A class had 32 students. Model the class using the file. Shade the square to answer each question and sketch your answer below.

- ***If 10 students liked math, what fraction of the class liked math?***

  Answer: $\frac{10}{32}$ or $\frac{5}{16}$; sketches should show either 32 with 10 shaded rectangles or 16 with 5 shaded rectangles.

- ***If 22 students were involved in sports, what fraction of the class was involved in sports?***

  Answer: $\frac{22}{32}$ or $\frac{11}{16}$

- ***If c of the class walked to school, how many students walked to school? How does the tiling activity help you answer the question?***

  Answer: 8 students walked to school. If you have 32 small rectangles, shading one column (or row) of the four columns, you will have v of the area, but one column has 8 rectangles so it is the same as $\frac{8}{32}$ of the area.

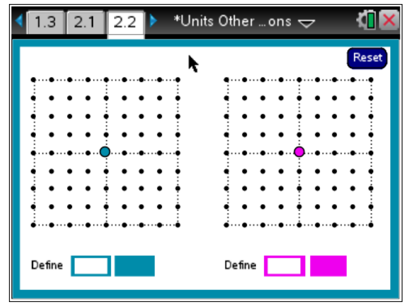
- ***If $\frac{3}{8}$ of the students were boys, how many boys were in the class?***

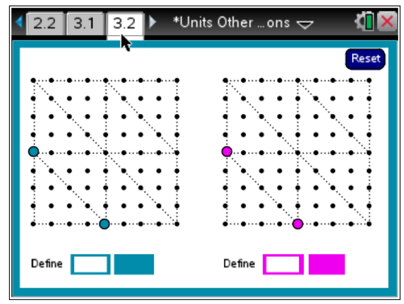
  Answer: There were 12 boys in the class. To determine this, shade three of the eight rows (columns), which shades 12 of the 32 rectangles.

Part 2, Page 2.2

Focus: Students will tile a rectangle to compare fractions.

Page 2.2 displays two larger rectangles, each of which can be partitioned into regions by using the **Define** commands at the bottom of the page in the same way as on the previous page. To reset one of the large rectangles, select the space for **Define** for that rectangle. Page 3.2 behaves the same way as page 2.2, but the shapes are triangular. **Reset** returns both of the larger rectangles to the default position.





Teacher Tip: As students work with or observe you working with the tiling activity, help them to relate the parts of the fractions to the defined area and the shaded area. Allow them to practice naming fractions illustrated in the larger rectangle and describing how a given fraction might be illustrated in the rectangle.

Class Discussion

Have students…	Look for/Listen for…
Move the circles to tile the large square on the left into 16 pieces and the large rectangle on the right into 8 pieces.	
• ***Border the entire large square on the right and eight of the small regions in the square on the left. Shade $\frac{1}{8}$ of each. How do the shaded areas of the two rectangles compare? Justify your thinking.***	Answer: The two $\frac{1}{8}$'s are different; the $\frac{1}{8}$ on the left has an area smaller than $\frac{1}{8}$ of the area on the right.
• ***Does $\frac{1}{8}$ have the same meaning in any context? Why or why not?***	Answer: $\frac{1}{8}$ will only have the same meaning if it refers to the same size unit or whole.

Class Discussion (continued)

Create a shape in each of the large squares that will show the following. Be ready to share your work and justify your reasoning.

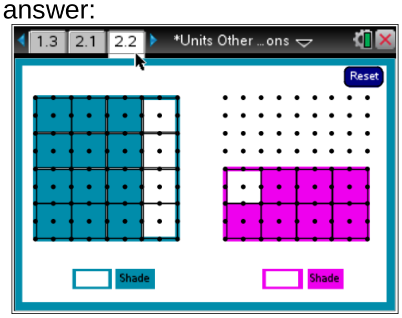
- ***$\frac{1}{2}$ does not always mean the same thing.***

Possible answer: In the same configuration as problem 4, shade $\frac{1}{2}$ of each.

- ✓ ***$\frac{3}{4}$ of one thing can be larger than $\frac{7}{8}$ of another.***

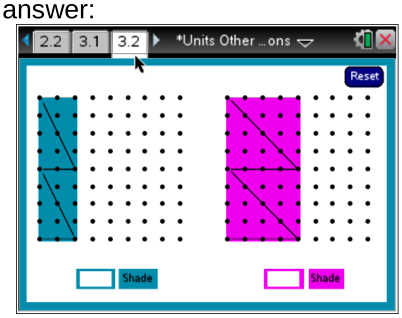
(Question #1 on the Student Activity Sheet.)

Possible answer:

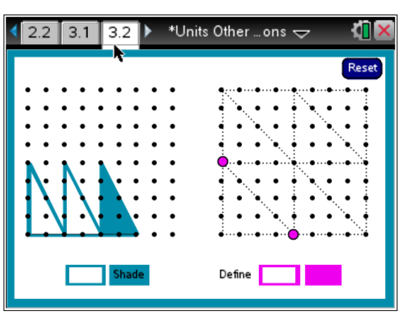


- ***On page 3,2, create a representation of 1 in the left region that is smaller than $\frac{3}{4}$ in the right region.***

Possible answer:



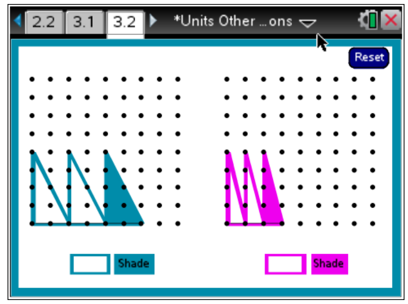
- ***Create the shape illustrated in the picture below in the region on the left. Create a representation in the region on the right to satisfy the following:***



Class Discussion (continued)

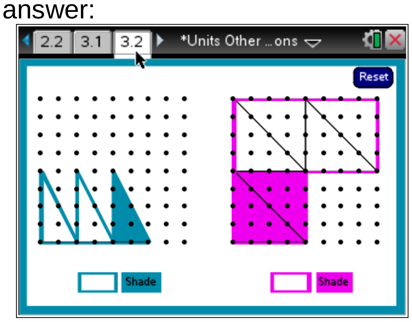
- ***$\frac{1}{3}$ with a smaller area than the one shown above.***

Possible answer:



- ***$\frac{1}{3}$ with a larger area than the one shown above.***

Possible answer:



Decide who is right and use any page from the tiling activity to support your thinking.

- ***Tom argued that half of his field was larger than half of his neighbor's field. Sam said that was impossible because $\frac{1}{2}$ is $\frac{1}{2}$.***

Possible answer: Tom is correct because the $\frac{1}{2}$ can be different depending on the size of the unit.

- ***Sallee said that a fair trade was $\frac{3}{4}$ of her share for $\frac{2}{3}$ of Marj's share. Marj said that was unfair.***

Possible answer: You cannot tell who was right because it depends on the size of the units or the wholes that are being compared; $\frac{2}{3}$ of a very big cake could be more than $\frac{3}{4}$ of a small one. If the shares came from the same size unit, then it would be unfair.

- ***Del said he ate half of a pizza. Suz said she ate half of a pizza but ate more than Del. Del said Suz was wrong.***

Possible answer: Same as the answer above with Tom and Sam. It depends on the size of the pizza.

Class Discussion (continued)

Using either pages 2.2 or 3.2, tile the large region on the left into 16 pieces and the other into 32 pieces.

- ***Create $\frac{8}{12}$ in the region on the left and $\frac{5}{12}$ in the region on the right. Explain whether you can find $\frac{5}{12}+\frac{8}{12}$. Justify your reasoning.***

Possible answer: You cannot add them because they represent fractions from different size units or different wholes. The unit fraction $\frac{1}{12}$ on the left represents a larger area than the unit fraction $\frac{1}{12}$ on the right.

- ***Reset the left region into 32 pieces. Create $\frac{8}{12}$ in the left and $\frac{5}{12}$ in the right. Explain whether you can find $\frac{5}{12}+\frac{8}{12}$. Justify your reasoning.***

Possible answer: You can add the fractions and get $\frac{13}{12}$ because both of the unit fractions represent $\frac{1}{12}$ of the same size unit or whole.

- ***Explain how your answer above might be used to find $\frac{2}{3}+\frac{5}{12}$.***

Possible answer: You can see that $\frac{2}{3}$ is the same as $\frac{8}{12}$ so $\frac{5}{12}+\frac{2}{3}$ is the same answer, $\frac{13}{12}$.

✓ ***Explain how you can use the tiling activity to find each of the following:***

a. $\frac{1}{2}+\frac{2}{5}$ ***b.*** $\frac{9}{10}-\frac{1}{5}$ ***c.*** $\frac{7}{8}-\frac{2}{3}$

(Question #2 on the Student Activity Sheet.)

Answer: Tile each large region into 24 or 32. Then border shapes that contain the same number of pieces as the product of the denominators. Find the equivalent fractions and add or subtract.

For a: $\frac{1}{2}+\frac{2}{5}=\frac{5}{10}+\frac{4}{10}=\frac{9}{10}$.

For c: $\frac{7}{8}-\frac{2}{3}=\frac{21}{24}-\frac{16}{24}=\frac{5}{24}$.

For b: $\frac{9}{10}-\frac{1}{5}=\frac{9}{10}-\frac{2}{10}=\frac{7}{10}$.

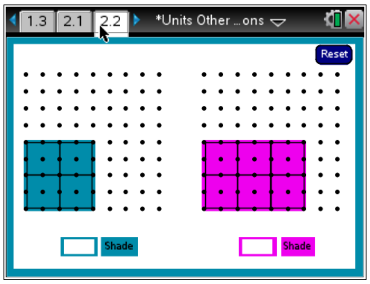
Class Discussion (continued)

Tile each of the large squares on page 2.2 into 16 small rectangles.

Border four of the small rectangles on the left. Consider these four small rectangles all together as a new "unit" and shade them. In the unit square at the right, create each of the following fractions using the unit in the large rectangle at the left. Show a sketch of your answers.

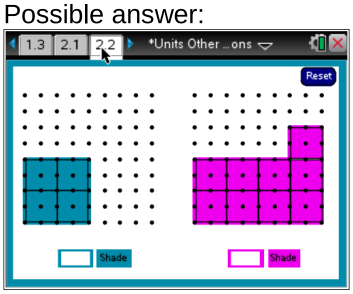
- $1\frac{1}{2}$

  Possible answer:

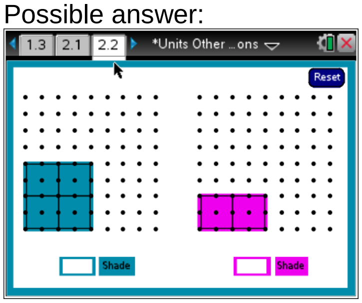
- $2\frac{1}{4}$

  Possible answer:

- $\frac{1}{2}$

  Possible answer:

Class Discussion (continued)

Tile each of the large squares on page 2.2 into 16 small rectangles. Change the border on the left to enclose three of the small rectangles. Shade them as a new "unit".

Identify the fraction of the unit if you

- ***Border two of the small rectangles on the right.***

  Answer: $\frac{2}{3}$

- ***Border four of the small rectangles on the right.***

  Answer: $1\frac{1}{3}$

- ***Border eight of the small rectangles on the right.***

  Answer: $2\frac{2}{3}$

Reset both regions with 16 small rectangles, again. In the left region, border and shade six of the small rectangles as your new unit. How many rectangles would you have to include on the right to represent each of the following in terms of the unit shaded on the left?

- $\boldsymbol{\frac{1}{2}}$

  Answer: 3

- $\boldsymbol{\frac{3}{2}}$

  Answer: 9

- $\boldsymbol{\frac{2}{3}}$

  Answer: 4

✓ ***Use your thinking from the problem above to find the answers to each of the following. Explain your reasoning.***

(Question #3 on the Student Activity Sheet.)

- $\boldsymbol{2 \times \frac{2}{5}}$

  Answer: $\frac{4}{5}$

- $\boldsymbol{4 \times \frac{3}{8}}$

  Answer: $\frac{12}{8}$ or $\frac{3}{2}$

Class Discussion (continued)

- $3 \times \frac{4}{8}$

Answer: $\frac{12}{8}$ or $\frac{3}{2}$

In each case you have a given number of the unit; for example, 2 sets of $\frac{2}{5}$ makes $\frac{4}{5}$.

A box contains 10 packages of instant oatmeal. Model this using the left unit square. Use the tiling on the right to find the number of packages in each.

- ***$\frac{1}{2}$ box***

  Answer: 5 packages
- ***$\frac{4}{5}$ box***

  Answer: 8 packages
- ***$\frac{13}{10}$ boxes***

  Answer: 13 packages
- ***$\frac{3}{2}$ boxes***

  Answer: 15 packages.

A pack of pencils contains 16 pencils. Model this using the unit square on the left.

Explain your thinking. (You may want to use the tiling on the right to help your reasoning.)

Look for sets of 4 in 16 for $\frac{1}{4}$ pack; look for sets of $\frac{3}{8}$ in 16 for $\frac{3}{8}$ pack. The answer to $\frac{1}{4}$ pack can be used to find the solution for $1\frac{1}{4}$ packs; the solution for $1\frac{1}{4}$ packs can be used to find the solution for $3\frac{3}{8}$ packs.

- ***$\frac{1}{4}$*** **pack**

  Answer: 4 pencils
- ***$\frac{3}{8}$*** **pack**

  Answer: 6 pencils
- ***$1\frac{1}{4}$*** **packs**

  Answer: 20 pencils
- ***$3\frac{3}{8}$*** **packs**

  Answer: 54 pencils.

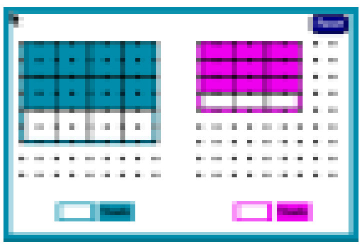
Sample Assessment Items

After completing the lesson, students should be able to answer the following types of questions. If students understand the concepts involved in the lesson, they should be able to answer the following questions without using the TNS activity.

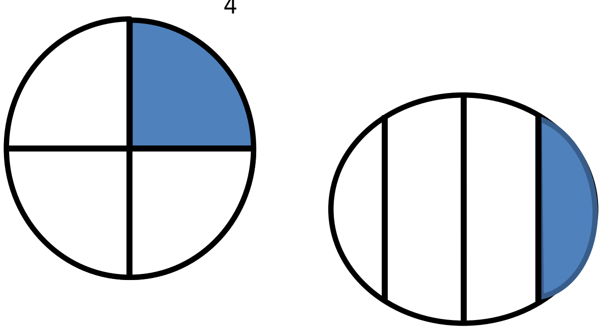
1. Jose ate $\frac{1}{2}$ of a pizza. Ella ate $\frac{1}{2}$ of another pizza. Jose said that he ate more pizza than Ella, but Ella said they both ate the same amount. Use words and pictures to show that Jose could be right. ***Answer: The pizzas could be of different size. Jose could have a large pizza and Ella a medium one.***

NAEP, 1992

2. Use the grids to illustrate when $\frac{2}{3}$ could represent a large r area than $\frac{3}{4}$. ***Possible answer:***



3. Present an argument that supports or does not support the statement that each representation below represents $\frac{1}{4}$.



Answer: The circle at the left is not partitioned into four regions with the same area; the shaded region is not $\boldsymbol{\frac{1}{4}}$ ***of the area of the circle. The circle at the right is partitioned into four regions with the same area so the shaded area is*** $\boldsymbol{\frac{1}{4}}$ ***of the area of the circle.***

4. A box of granola bars contains 18 bars. How many bars will be in $\frac{2}{3}$ of the box? ***Answer: 12 bars***

5. In a class of 24 students, 18 are girls. What fraction of the class are girls? ***Answer:*** $\boldsymbol{\frac{18}{24} = \frac{3}{4}}$

Student Activity solutions

Vocabulary

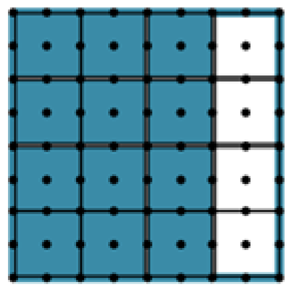
equivalent fractions: fractions that are located at the same point on the number line

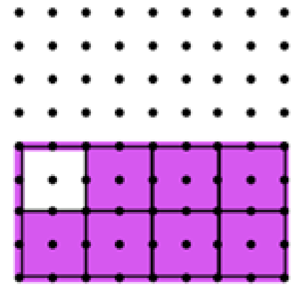
unit fraction: a fraction where the numerator is 1

In this activity, you will use grids and tiling to help solve fraction problems.

1. Draw a shape in each of the large squares that will show that $\frac{3}{4}$ of one thing can be larger than $\frac{7}{8}$ of another.

Answer:





2. Explain how you can use tiling to find each of the following:

a. $\frac{1}{2}+\frac{2}{5}$

b. $\frac{9}{10}-\frac{1}{5}$

c. $\frac{7}{8}-\frac{2}{3}$

Answer: For a and c, tile each large region into 24 or 32. Then border shapes that contain the same number of pieces as the product of the denominators. Find the equivalent fractions and add or subtract. For a, $\frac{1}{2}+\frac{2}{5}=\frac{5}{10}+\frac{4}{10}=\frac{9}{10}$; for c, $\frac{7}{8}-\frac{2}{3}=\frac{21}{24}-\frac{16}{24}=\frac{5}{24}$; for b, $\frac{9}{10}-\frac{1}{5}=\frac{9}{10}-\frac{2}{10}=\frac{7}{10}$.

3. Solve each of the following. Explain your reasoning.

a. $2 \times \frac{2}{5}$ b. $4 \times \frac{3}{8}$ c. $3 \times \frac{4}{8}$

Answers: a. $\frac{4}{5}$ b. $\frac{12}{8}$ or $\frac{3}{2}$ c. $\frac{12}{8}$ or $\frac{3}{2}$. In each case you have a given number of the unit; for example, 2 sets of $\frac{2}{5}$ makes $\frac{4}{5}$.

4. A box contained 12 packages of trail mix. Use tiling to find the number of packages in each.

a. $\frac{1}{2}$ box c. $\frac{14}{12}$ boxes

b. $\frac{2}{3}$ box d. $\frac{4}{2}$ boxes

Answers: a. 6 packages; b. 8 packages; c. 14 packages; d. 24 packages.