Building Concepts: Comparing Units

Lesson Overview

This TI-Nspire™ lesson helps students to understand that the size of a fraction will vary with the scale used to define a unit fraction. For example, $\frac{1}{2}$ of an inch is a different length than $\frac{1}{2}$ of a foot.

Without knowing what each refers to, $\frac{1}{2}$ of one thing can be the same as, more than, or less than $\frac{1}{2}$ of another.

Learning Goals

Students should understand and be able to explain each of the following:

1. Fractions do not necessarily come from the same whole, which can make a difference in what two fractions represent in terms of length or area;
2. Fractions that represent different wholes cannot be added or subtracted;
3. Fractions can only be compared wher they represent the same whole.

Prerequisite Knowledge

Comparing Units is the last lesson (15) in the series of lessons that explore fractions. This lesson builds on the concepts explored in the previous lessons *Fractions and Unit Squares* and *Units Other Than a Unit Square.* Prior to working on this lesson students should understand:

- the concept of equivalent fractions.
- how to compare fractions.
- how to add and subtract fractions.

Vocabulary

- **scalar multiple:** the given amount of a specified quantity. For example, when you have ***a*** amount of quantity ***b***, ***a*** is the scalar multiple of ***b***.

Lesson Pacing

This lesson should take 50 minutes to complete with students, though you may choose to extend, as needed.

Lesson Materials

- Compatible TI Technologies:

  TI-Nspire CX Handhelds, TI-Nspire Apps for iPad®, TI-Nspire Software
- Comparing Units_Student.pdf
- Comparing Units_Student.doc
- Comparing Units.tns
- Comparing Units_Teacher Notes
- To download the TI-Nspire activity (TNS file) and Student Activity sheet, go to http://education.ti.com/go/buildingconcepts.

Class Instruction Key

The following question types are included throughout the lesson to assist you in guiding students in their exploration of the concept:

Class Discussion: Use these questions to help students communicate their understanding of the lesson. Encourage students to refer to the TNS activity as they explain their reasoning. Have students listen to your instructions. Look for student answers to reflect an understanding of the concept. Listen for opportunities to address understanding or misconceptions in student answers.

✓ **Student Activity Sheet**: The questions that have a check-mark also appear on the Student Activity Sheet. Have students record their answers on their student activity sheet as you go through the lesson as a class exercise. The student activity sheet is optional and may also be completed in smaller student groups, depending on the technology available in the classroom. A (.doc) version of the Teacher Notes has been provided and can be used to further customize the Student Activity sheet by choosing additional and/or different questions for students.

Bulls-eye Question: Questions marked with the bulls-eye icon indicate key questions a student should be able to answer by the conclusion of the activity. These questions are included in the Teacher Notes and the Student Activity Sheet. The bulls-eye question on the Student Activity sheet is a variation of the discussion question included in the Teacher Notes.

Mathematical Background

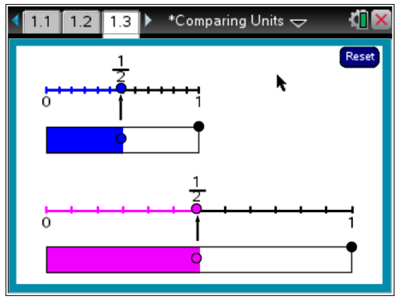
This TI-Nspire™ lesson helps students to understand that the size of a fraction will vary with the scale used to define a unit fraction. For example, $\frac{1}{2}$ of an inch is a different length than $\frac{1}{2}$ of a foot. The idea of what represents the whole is critical to developing an understanding of fractions. Without knowing what each refers to, $\frac{1}{2}$ of one thing can be the same as, more than, or less than $\frac{1}{2}$ of another (e.g., $\frac{1}{2}$ of a foot is larger than $\frac{1}{2}$ of an inch but shorter than $\frac{1}{2}$ of a meter.) Thus, students should understand the importance of recognizing the scale or size of the unit when working with fractions. For instance, adding fractions representing different units can produce nonsense answers: $\frac{1}{2}$ of a foot plus $\frac{1}{2}$ of an inch is not 1 whole of anything.

Analyzing the relationship between one fraction and another when they come from different wholes can be misleading. Consider $\frac{1}{4}$ of a quantity is not conceptually the same as looking at the fraction $\frac{1}{4}$ on the number line or in an area model. When you have ***a*** amount of a quantity ***b***, ***a*** is what is called a scalar multiple of ***b***.

Part 1, Page 1.3

Focus: Students will compare fractions from different-sized wholes.

Page 1.3 displays two number lines with corresponding bars that represent two different sized wholes. Dragging the black dot below either number line will change the length of the number line and dragging the blue or pink dot on the number lines will change the corresponding fraction bars.



TI-Nspire Technology Tips

Students may find it easier to use the tab key to toggle between objects and then use the arrow keys to move or change their selections.

To reset the page, select **Reset** in the upper right corner.

Teacher Tip: Be sure students understand how the interaction with the number lines supports the mathematics. Discuss scale with students. Have them provide examples of different types of scales and compare them.

Give students time to repeat the activity before asking them a focused set of questions. This will help them internalize the concept of the different sized wholes and fractions. Encourage students to explain their reasoning for the answers.

Class Discussion

Have students…	Look for/Listen for…
Suppose you have two different sizes of the same candy bar; the giant and the regular size. Consider the giant bar to be twice the size of the regular size bar. On page 1.3 of the activity, use the two number lines to represent the candy bars.	
• ***Would it be fair to cut each bar in half and share the four pieces among four people? Why or why not?***	Answer: It would not be fair to share the candy bars in this way because $\frac{1}{4}$ of the giant bar is larger than $\frac{1}{4}$ of the regular bar.

Class Discussion (continued)

✓ ***If you had one of each bar, how much candy would you have in terms of the "giant" bar? In terms of the "regular" bar?*** (Question #1 on the Student Activity sheet.)

Answer: You would have an amount of candy the same as $1\frac{1}{2}$ of the giant bars and 3 of the regular bars.

- ***Drag one of the dots to show how big the candy company could make each bar so sharing both a giant and a regular candy bar amongst four people would be fair. Sketch your answer below.***

Answer: Any solution that shows the two bars the same size. Or, if the regular bar was $\frac{1}{3}$ the size of the giant bar, you could divide the giant bar into 3 pieces and each person would have $\frac{1}{3}$ of the giant bar.

Use the number lines to answer each of the following and explain your reasoning in each case.

- ***If the regular candy bar was three fourths of the giant candy bar, how much more candy would the person with the giant candy bar have? Explain your thinking.***

Answer: The person with the giant candy bar would have $\frac{1}{4}$ more than the person with the regular-sized candy bar because $\frac{3}{4}$ and $\frac{1}{4}$ make one whole giant candy bar.

- ***If the giant bar was three times the size of the regular bar, how much candy would you have if you had both of the bars? Explain how you found your answer.***

Answer: In terms of the giant candy bar, you would have $1\frac{1}{3}$ of the giant bar because you would have the whole giant bar and $\frac{1}{3}$ as much of the giant bar. In terms of the regular candy bar, you would have 4 of those candy bars because the giant candy bar is the same as 3 regular bars.

- ***Sam said that if the giant candy bar is twice as big as the regular candy bar, $\frac{1}{12}$ of the larger candy bar would be the same amount of candy as $\frac{1}{6}$ of the smaller candy bar. Do you agree with Sam? Why or why not? (You may use the number lines to support your thinking.)***

Answer: Sam is correct because you would need 2 copies of $\frac{1}{12}$ of the smaller candy bar to have one copy of $\frac{1}{12}$ of the larger candy bar because the smaller bar is $\frac{1}{2}$ of the larger.

Class Discussion (continued)

✓ ***Find sizes for regular and giant bars that could be divided equally***
(Question #2 on the Student Activity Sheet.)

- ***among six people***

Possible answer: The regular bar could be $\frac{1}{2}$ of the giant bar. Then divide the regular bar into 2 pieces and the giant bar into four pieces. Each person would get $\frac{1}{4}$ of the giant candy bar.

- ***among twelve people***

Possible answers: If the regular candy bar was $\frac{1}{2}$ of the giant candy bar, you could divide the larger candy bar into 8 pieces and the smaller candy bar into 4 pieces. Each person would have $\frac{1}{8}$ of the larger candy bar. Another possible answer would be to divide each of the bars into 12 pieces and give everyone $\frac{1}{12}$ of each bar.

On the number lines in the activity, set one bar to half the size of the other. Which is greater in total length for each of the following? Explain your reasoning in each case.

✓ $\frac{2}{3}$ ***of the smaller or*** $\frac{1}{2}$ ***of the larger***
(Question #3 on the Student Activity sheet.)

Answer: $\frac{1}{2}$ of the larger bar is bigger because $\frac{2}{3}$ of the smaller is only $\frac{1}{3}$ of the larger.

- $\frac{5}{12}$ ***of the larger or*** $\frac{11}{12}$ ***of the smaller***

Answer: $\frac{11}{12}$ of the smaller bar is greater because $\frac{11}{12}$ is equivalent to $\frac{11}{24}$ on the larger bar and $\frac{5}{12}$ is the same as $\frac{10}{24}$. $\frac{11}{24} > \frac{10}{24}$.

$\frac{1}{2}$ ***of the smaller or*** $\frac{1}{4}$ ***of the larger***

Answer: They are the same size because $\frac{1}{2}$ of the smaller bar is $\frac{1}{4}$ of the larger bar.

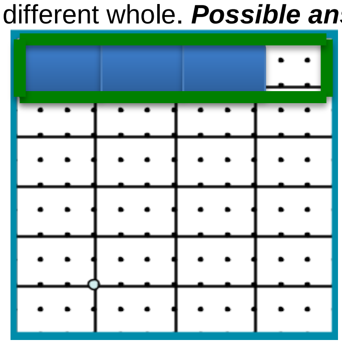
Sample Assessment Items

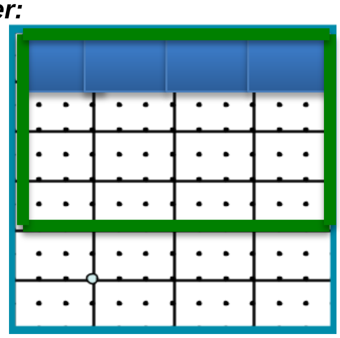
After completing the lesson, students should be able to answer the following types of questions. If students understand the concepts involved in the lesson, they should be able to answer the following questions without using the TNS activity.

1. Claire and Sarah each had $\frac{1}{12}$ of a cake. Claire claimed she had more cake than Sarah. Do you think she could be right? Why or why not? ***Answer: If they had pieces cut from the same cake, Claire is wrong. But, if Claire had a piece from a larger cake than Sarah's, then Claire's $\frac{1}{12}$ could be larger.***

2. Petre had $\frac{1}{3}$ of a 6-inch long candy bar and Sande had $\frac{1}{2}$ of 4-inch candy bar. Who had the most candy? ***Answer: Petre had 2 inches of his bar because $\frac{1}{3}$ of 6 is 2. Sande had 2 inches of her candy bar because $\frac{1}{2}$ of 4 is 2. So they had the same amount of candy.***

3. Use the grids to shade two fractions that would illustrate $\frac{3}{4}$ of one whole is smaller than $\frac{1}{3}$ of a different whole. ***Possible answer:***





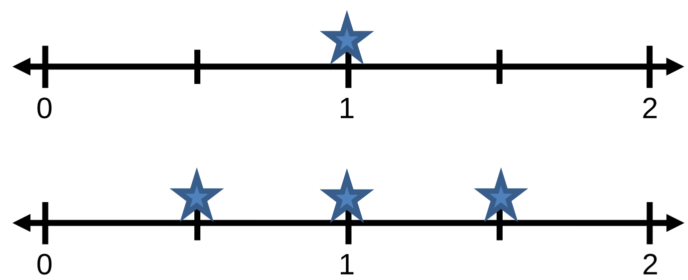
Student Activity solutions

Vocabulary

scalar multiple: the given amount of a specified quantity. For example, when you have ***a*** amount of quantity ***b***, ***a*** is the scalar multiple of ***b***.

In this activity, you will compare two fractions that come from different wholes.

1. Suppose you have two different sizes of the same candy bar; the giant and the regular size. Consider the giant bar to be twice the size of the regular size bar. Use the two number lines to represent the candy bars. If you had one of each bar, how much candy would you have in terms of the giant bar? In terms of the regular bar?



Answer: You would have an amount of candy the same as $1\frac{1}{2}$ of the larger bar and 3 of the smaller bars.

2. Find sizes for regular and giant bars that could be divided equally among six people.

Possible answer: The regular bar could be $\frac{1}{2}$ of the giant bar. Then divide the regular bar into 2 pieces and the giant bar into four pieces. Each person would get $\frac{1}{4}$ of the giant candy bar.

3. On the number lines in the activity, set one bar to half the size of the other. Which is greater in total length: $\frac{2}{3}$ of the smaller or $\frac{1}{2}$ of the larger? Explain your reasoning.

Answer: $\boldsymbol{\frac{1}{2}}$ ***of the larger bar is bigger because*** $\boldsymbol{\frac{2}{3}}$ ***of the smaller is only*** $\boldsymbol{\frac{1}{3}}$ ***of the larger.***

4. Jenna drew two number lines, each with a bar above it. She set one bar to one third the size of the other. Which is greater in total length, $\frac{1}{3}$ of the smaller bar or $\frac{1}{9}$ of the larger bar? Explain your reasoning.

Answer: They are the same size. One bar is three times the size of the other, so $\boldsymbol{\frac{1}{3}}$ ***of the smaller is equal to*** $\boldsymbol{\frac{1}{9}}$ **of the larger number line.**