Building Concepts: Adding and Subtracting Fractions with Like Denominators

TEACHER NOTES

Lesson Overview

This TI-Nspire™ lesson uses interactive number lines to investigate addition and subtraction of fractions that refer to the same whole (i.e. have the same denominator). Students may reason about the process of adding or subtracting fractions with "like" denominators as 1) decomposing each fraction into unit fractions and counting the total number of unit fractions that would be marked on the number line; or 2) the series of the lengths from 0 represented by each fraction, when those segments are joined end to end.

Subtraction can be thought of as finding the missing addend or as taking away the number of copies of $\frac{a}{b}$ from the number of copies of $\frac{c}{b}$. Adding two fractions with denominators of $\frac{1}{b}$ is the same as joining the first segment $\frac{a}{b}$ end to end with the second segment $\frac{c}{b}$.

Learning Goals

1. Understand that a fraction that is not a unit fraction can be decomposed into a sum of unit fractions.
2. Explain that two fractions with common denominators can be added (or subtracted) by decomposing each fraction into unit fractions and counting the total (or the difference in the) number of unit fractions.
3. Recognize that if two fractions represent the same whole and have the same denominator, their sum is represented by joining end to end the segment of length for one fraction to the segment of the length represented by the second.
4. Find the difference of two fractions representing the same whole with common denominators by subtracting the length of the segment for one fraction from the length represented by the sum of the two fractions.
5. Understand that the sum (or difference) of two fractions with a common denominator in general, can be found by summing (or subtracting) the numerators of each fraction $\left(i.e., \frac{a}{b} \pm \frac{c}{b} = \frac{a \pm c}{b}\right)$.

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Prerequisite Knowledge

Adding and Subtracting Fractions with Like Denominators is the fifth lesson in a series of activities that explore fractions and build on the concepts explored in the previous lessons. Students should be familiar with the terms *unit fraction*, *equivalent fractions*, and *improper fraction* covered in earlier lessons. Prior to working on this lesson, students should understand:

- how the numerator and denominator give a fraction its value.
- the concept of fractions on a number line.
- the concept of adding and subtracting whole numbers.

Vocabulary

The following <u>properties of addition</u> are true for all real numbers.

- commutative property for addition: a + b = b + a
- associative property for addition: (a + b) + c = a + (b + c)
- inverse property for addition: a + (– a) = – a + a = 0
- identity property for addition: a + 0 = a

Lesson Pacing

This lesson contains multiple parts and can take 50–90 minutes to complete with students, though you may choose to extend, as needed.

Lesson Materials

- Compatible TI Technologies:

  TI-Nspire CX Handhelds, TI-Nspire Apps for iPad®, TI-Nspire Software
- Adding and Subtracting Fractions with Like Denominators_Student.pdf
- Adding and Subtracting Fractions with Like Denominators_Student.doc
- Adding and Subtracting Fractions with Like Denominators.tns
- Adding and Subtracting Fractions with Like Denominators_Teacher Notes
- To download the TI-Nspire activity (TNS file) and Student Activity sheet, go to http://education.ti.com/go/buildingconcepts.

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Class Instruction Key

The following question types are included throughout the lesson to assist you in guiding students in their exploration of the concept:

Class Discussion: Use these questions to help students communicate their understanding of the lesson. Encourage students to refer to the TNS activity as they explain their reasoning. Have students listen to your instructions. Look for student answers to reflect an understanding of the concept. Listen for opportunities to address understanding or misconceptions in student answers.

✓ **Student Activity Sheet**: The questions that have a check-mark also appear on the Student Activity Sheet. Have students record their answers on their student activity sheet as you go through the lesson as a class exercise. The student activity sheet is optional and may also be completed in smaller student groups, depending on the technology available in the classroom. A (.doc) version of the Teacher Notes has been provided and can be used to further customize the Student Activity sheet by choosing additional and/or different questions for students.

Bulls-eye Question: Questions marked with the bulls-eye icon indicate key questions a student should be able to answer by the conclusion of the activity. These questions are included in the Teacher Notes and the Student Activity Sheet. The bulls-eye question on the Student Activity sheet is a variation of the discussion question included in the Teacher Notes.

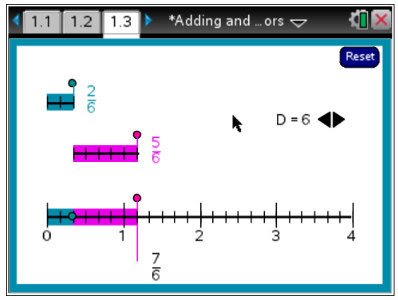
Mathematical Background

This TI-Nspire™ activity uses interactive number lines to investigate addition and subtraction of fractions that refer to the same whole (i.e. have the same denominator). Students may reason about the process of adding or subtracting fractions with "like" denominators as 1) decomposing each fraction into unit fractions and counting the total number of unit fractions that would be marked on the number line; or 2) the series of the lengths from 0 represented by each fraction, when those segments are joined end to end. Subtraction can be thought of as finding the missing addend (i.e. for $\frac{c}{b}-\frac{a}{b}$, what would you add to $\frac{a}{b}$ to obtain $\frac{c}{b}$?) or as taking away the number of copies of $\frac{a}{b}$ from the number of copies of $\frac{c}{b}$.

Students should be familiar with the concept of fraction as a point on a number line before they begin this lesson. Understanding $\frac{a}{b}$ as ***a*** copies of $\frac{1}{b}$ marked on the number line beginning from 0 suggests that adding two fractions with denominators of $\frac{1}{b}$ is the same as joining the first segment $\frac{a}{b}$ end to end with the second segment $\frac{c}{b}$. The lesson can also be used to investigate the properties for addition (commutative, associative, inverse, and identity).

Part 1, Page 1.3

Focus: Students will add fractions with the same denominator on a number line.



In this activity, interactive number lines are used to help students investigate addition of fractions. Three number lines are displayed: the two number lines at the top of the page represent the fractions to be added, and the bottom number line represents the sum of the two fractions. The denominators of the fractions can be changed using the arrows to the right. Moving the dots on any of the number lines changes the numerators of the corresponding fractions. As the number lines at the top of the page are changed, the corresponding fractional part on the number line below is also changed.

TI-Nspire Technology Tips

Students may find it easier to use the tab key to toggle between objects and then use the arrow keys to move or change their selections.

To reset the page, select **Reset** in the upper right corner.

Teacher Tip: Use the interactive number line to explain why the sum of two fractions has the same denominator as the addends. Guide students in understanding that the denominators of the fractions are not added. For example, adding $\frac{2}{5}$ and $\frac{1}{5}$ results in $\frac{3}{5}$, not $\frac{3}{10}$.

Lead students in a discussion of the sum of the fractions, as shown on the number lines. Encourage students to explain their reasoning for the answers they provide.

Class Discussion

Have students…	Look for/Listen for…
Add $\frac{3}{5}$ to $\frac{4}{5}$ and explain in terms of unit fractions why your answer makes sense.	Sample answer: You are counting the number of $\frac{1}{5}$ s all together. You have 3 copies of $\frac{1}{5}$ and 4 copies of $\frac{1}{5}$ all on the same number line so you should have 7 copies of $\frac{1}{5}$ or $\frac{7}{5}$.

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Class Discussion (continued)

- ***What fraction could you add to $\frac{4}{7}$ to get a fraction greater than 1?***

  Answer: Any fraction that was larger than $\frac{3}{7}$.

- ✓ ***If the sum of two fractions with denominators of 4 is $\frac{7}{4}$, what could be the fractions that were added?***

  (Question #1 on the Student Activity sheet.)

  Answer: $\frac{1}{4}+\frac{6}{4}$; $\frac{2}{4}+\frac{5}{4}$; *or* $\frac{3}{4}+\frac{4}{4}$ (students may give answers in reduced form).

- ***Decompose $\frac{7}{11}$ into two fractions with the same denominators so the numerators are consecutive whole numbers (one right after another.)***

  Answer: $\frac{3}{11}+\frac{4}{11}$.

- ***Can you decompose $\frac{21}{11}$ into two fractions with the same denominators whose numerators are both even? Why or why not?***

  Answer: It is impossible because the numerators would have to sum to an odd number and that can only happen if you add an odd number to an even number.

- ***Decompose $\frac{35}{12}$ into a whole number and a fraction.***

  Answer: $\frac{12}{12}+\frac{23}{12}$; $\frac{24}{12}+\frac{11}{12}$.

Have students…

Explain whether each equation is true or not true using the interactive number line to support your reasoning.

- $\frac{7}{8}+\frac{5}{8} = \frac{5}{8}+\frac{7}{8}$

Look for/Listen for…

Answer: True because you are just changing the order in which the numbers are added (commutative property of addition).

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Class Discussion (continued)

- $\left(\frac{3}{8}+\frac{1}{8}\right)+\frac{5}{8} = \frac{3}{8}+\left(\frac{1}{8}+\frac{5}{8}\right)$

  Answer: True because of the associative property of addition; regrouping does not change the answer.

- $\frac{3}{8}+\frac{3}{8} = 3\left(\frac{2}{8}\right)$

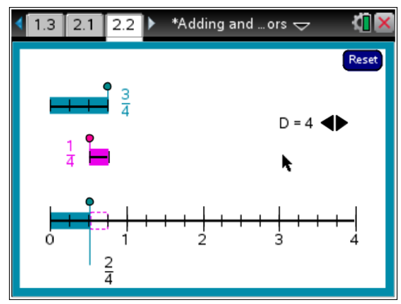
  Answer: True. $\frac{1}{8}+\frac{1}{8}=\frac{2}{8}$ and 3 sets of $\frac{2}{8}$ would be $\frac{6}{8}$ (Distributive property).

- $\frac{3}{8}+\frac{3}{8} = \frac{3}{4}$

  Answer: True because $\frac{6}{8}=\frac{3}{4}$.

Part 2, Page 2.2

Focus: Students will use a number line to subtract fractions with the same denominator.



The top number line represents the total (the number from which a fraction will be subtracted). The second number line represents the fraction to be subtracted from the total. The arrows and dots are used the same way as those on page 1.2 of the activity. Note that it is not possible to subtract a larger fraction from a smaller one using these number lines. To reset the page, select **Reset** in the upper right corner.

> **Teacher Tip:** As students observe what is happening in this file, help them focus on how subtraction is represented on the number line. Lead students to understand that the quantity shown on the bottom number line represents the difference in the subtraction. Give students sufficient opportunities to observe fraction subtractions that include improper fractions as well as standard fractions.

Class Discussion

Have students…

✓ ***Find $\frac{7}{10}-\frac{2}{10}$. Explain your answer using unit fractions.***

(Question #2 on the Student Activity sheet.)

Look for/Listen for…

Answer: 7 copies of $\frac{1}{10}$ minus 2 copies of $\frac{1}{10}$ will give 5 copies of $\frac{1}{10}$ so the answer is $\frac{5}{10}$. This can also be written as $\frac{1}{2}$.

- If the sum of two fractions is $\frac{8}{11}$ and one fraction is $\frac{5}{11}$, what is the other?

  Answer: $\frac{3}{11}$.

Class Discussion (continued)

> **Teacher Tip:** For each part in the following question have students illustrate the subtraction on number lines they have drawn. Then have them model the subtraction on the interactive number lines.

Suppose you have a board $\frac{11}{3}$ feet long.

Explain your reasoning in each of the following:

- ***If you saw off $\frac{4}{3}$ feet, will the remaining board be more or less than 3 feet long?***

  Answer: $\frac{11}{3}-\frac{4}{3}=\frac{7}{3}$, which is less than 3 feet (3 ft would be represented by $\frac{9}{3}$ ft.)

- ***What will you have to saw off to get a board 2 feet long?***

  Answer: Starting with $\frac{11}{3}$ ft. of board, in order to get 2 ft. (or $\frac{6}{3}$ ft.), you would have to saw off $\frac{5}{3}$ ft.

- ***How many thirds will you have to saw off to get a board that is less than $\frac{1}{2}$ foot long?***

  Possible answer: 10; You would saw off $\frac{10}{3}$ ft.

Use the interactive number line to solve the following questions:

- ***What is the largest positive whole number you can subtract from $\frac{23}{8}$? What is the result?***

  Answer: The largest is $\frac{16}{8}$ or 2. The result will be $\frac{7}{8}$.

- ***What is the smallest fraction with a denominator of 12 you can subtract from $\frac{17}{12}$ to get a fraction less than 1?***

  Answer: $\frac{6}{12}$ or $\frac{1}{2}$.

Class Discussion (continued)

- ***Sami argued that*** $\frac{7}{4}-\frac{6}{4}=\frac{1}{0}$***. What would you say to Sami?***

Possible answer: Sami forgot that you have to keep the unit fraction. You subtract the numerators, which tell you how many copies of the unit fraction you have.

- ***Describe how you can find*** $\frac{5}{12}+\frac{13}{12}+\frac{17}{12}$ ***in at least two different ways.***

Possible answer: You can add the $\frac{13}{12}$ and the $\frac{5}{12}$ to get $\frac{18}{12}$ and then add in the $\frac{17}{12}$ to get $\frac{35}{12}$ or you can add the last two terms first, $\frac{17}{12}+\frac{13}{12}$ to get $\frac{30}{12}$ and then add in the $\frac{5}{12}$ to get $\frac{35}{12}$.

Identify the following as true or false.

- $\frac{11}{8}-\frac{3}{8}-\frac{1}{8} = \frac{7}{8}$

  Answer: True.

- $\frac{11}{8}-\left(\frac{3}{8}-\frac{1}{8}\right)= \frac{7}{8}$

  Answer: False. It should be $\frac{9}{8}$.

- $\left(\frac{11}{8}-\frac{3}{8}\right)-\frac{1}{8} = \frac{7}{8}$

  Answer: True.

- $\frac{11}{8}-\left(\frac{3}{8}+\frac{1}{8}\right)=\frac{7}{8}$

  Answer: True.

- $\frac{3}{2}+\left(\frac{1}{2}+\frac{5}{2}\right)=\frac{9}{2}$

  Answer: True.

- $\left(\frac{3}{2}+\frac{1}{2}\right)+\frac{5}{2} = \frac{9}{2}$

  Answer: True.

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Class Discussion (continued)

Have students…

- ***Look at your work in the previous problem. Explain whether it seems to make a difference where the parentheses are located when you add and subtract three terms.***

Look for/Listen for…

Answer: When you grouped off the last two terms that were subtracted in $\frac{11}{8}-\left(\frac{3}{8}-\frac{1}{8}\right)=\frac{7}{8}$, you got the wrong answer. You got the right answer when you added the two you wanted to subtract and then did the subtracting like in $\frac{11}{8}-\left(\frac{3}{8}+\frac{1}{8}\right)=\frac{7}{8}$. The parentheses do not make a difference when adding, but they do make a difference when subtracting.

Find the missing value in each equation

- $\frac{x}{9}+\frac{2}{9}=\frac{13}{9}$

  Answer: $x = 11$.

- $\frac{13}{9}-\frac{x}{9}=\frac{2}{9}$

  Answer: $x = 11$.

- $\frac{13}{9}-\frac{2}{9}=\frac{x}{9}$

  Answer: $x = 11$.

- $\frac{19}{10}-\frac{2\cdot x}{10}=\frac{3}{10}$

  Answer: $x = 8$.

- $\frac{19}{10}+\frac{2\cdot x}{10}=\frac{23}{10}$

  Answer: x = 2.

- $\frac{21}{11}-\frac{x}{11}=0$

  Answer: $x = 21$.

Class Discussion (continued)

Answer each of the following and explain your thinking.

- ***Bets added $\frac{3}{4}$ cup of milk to a recipe, and then added another $\frac{5}{4}$ cups of milk. How much milk did she add all together?***

  Answer: She added $\frac{8}{4}$ or 2 cups of milk.

- ✓ ***Tomas ran $\frac{5}{12}$ of a mile. How much farther would he have to run to complete two miles?***

  (Question #3 on the Student Activity sheet.)

  Answer: He would have to run another $\frac{19}{12}$ miles for a total of 2 miles.

- ***Look at the divisors and the original numerator and denominator carefully. What observations can you make about the relationship in each of the problems above?***

  Answer: If the numerator and denominator of a fraction have a common factor, you can find an equivalent fraction that has a smaller numerator and denominator. If they don't have a common factor, you cannot.

- ***Suppose you had to reduce $\frac{24}{16}$. Think about your answers above and make a conjecture about the divisors that will work. Check your thinking using the number line.***

  Answer: 2, 4, and 8

- ***Suppose you want to reduce $\frac{25}{10}$. Why is it not useful to try 2 as a divisor?***

  Answer: Because you need a number that is a factor of both the numerator and denominator. 2 is only a factor of the denominator.

Sample Assessment Items

After completing the lesson, students should be able to answer the following types of questions. If students understand the concepts involved in the lesson, they should be able to answer the following questions without using the TNS activity.

1. Which explanation best describes why you need common denominators to add fractions?
   a. You need common denominators so you can combine parts of the fractions.
   b. To add fractions, you add the numerators and add the denominators, and it is easier to add the denominators when they are the same.
   c. If the denominators are not the same, the fractions could overlap.
   d. If you have *a* many copies of $\frac{1}{b}$ and *c* many copies of a different unit fraction like $\frac{1}{d}$, you cannot copy the two sets of fractions on the same number line at the same time because the partitions are different.

   Answer: d.

2. $\frac{4}{6} - \frac{1}{6} =$

   a. 3 b. $\frac{3}{6}$ c. $\frac{3}{0}$ d. $\frac{5}{6}$

   NAEP 2005, Grade 4

   Answer: b.

3. Put the values into the blanks to make true statements. You may use a value more than once.

   Possible answer: 2a could be $\frac{19}{6} - \frac{1}{6} = \frac{18}{6} = 3$ ***; 2d could be*** $\frac{6}{6} - \frac{1}{6} = \frac{5}{6}$.

4. In how many ways can you decompose $\frac{5}{4}$ into the sum of two fractions with a denominator of 4?

   Answer: three ways: $\frac{0}{4} + \frac{5}{4}$; $\frac{1}{4} + \frac{4}{4}$; $\frac{2}{4} + \frac{3}{4}$.

5. Put the values into the blanks to make true statements. You may use a value more than once.

$\frac{1}{3}, \frac{2}{3}, \frac{3}{3}, \frac{4}{3}, \frac{5}{3}, \frac{7}{3}, \frac{8}{3}, 3$

a. $\boxed{} - \frac{7}{3} = \boxed{}$ ***Answer:*** $\frac{8}{3} - \frac{7}{3} = \frac{1}{3}$

b. $\frac{13}{3} - \boxed{} = \boxed{}$ ***Answer:*** $\frac{13}{3} - \frac{8}{3} = \frac{5}{3}$

c. $\boxed{} - \frac{5}{3} = \boxed{}$ ***Answer:*** $\frac{7}{3} - \frac{5}{3} = \frac{2}{3}$

d. $\boxed{} + \boxed{} = \frac{7}{3}$ ***Answer:*** $\frac{2}{3} + \frac{5}{3} = \frac{7}{3}$

e. $\boxed{} - \boxed{} = \frac{4}{3}$ ***Answer:*** $\frac{8}{3} - \frac{4}{3} = \frac{4}{3}$

6. A pizza was cut into 10 pieces. Amy ate $\frac{1}{5}$ of the pieces. What fraction of the pieces was left?

Answer: $\frac{8}{10}$ ***or*** $\frac{4}{5}$.

Student Activity solutions

Vocabulary

The following <u>properties of addition</u> are true for all real numbers.

Commutative Property:

a + b = b + a

Associative Property:

(a + b)+ c = a + (b + c)

Inverse Property:

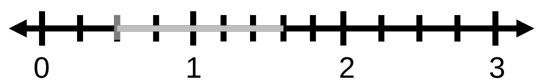
a + (– a) = – a + a = 0

Identity Property:

a + 0 = a

In this activity you will add and subtract fractions using a number line.

1. If the sum of two fractions with denominators of 4 is $\frac{7}{4}$, what could be the fractions that were added? Draw the fractions on the number line to show the addition.

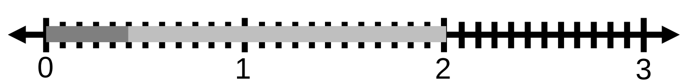


Answer: $\frac{1}{4}+\frac{6}{4}$; $\frac{2}{4}+\frac{5}{4}$; ***or*** $\frac{3}{4}+\frac{4}{4}$ ***(students may give answers in reduced form).***

2. Find $\frac{7}{10}-\frac{2}{10}$. Explain your answer using unit fractions.

Answer: 7 copies of $\frac{1}{10}$ ***minus 2 copies of*** $\frac{1}{10}$ ***will give 5 copies of*** $\frac{1}{10}$ ***so the answer is*** $\frac{5}{10}$***. This can also be written as*** $\frac{1}{2}$***.***

3. Tomas ran $\frac{5}{12}$ of a mile. How much farther would he have to run to complete two miles?



Answer: He would have to run another $\frac{19}{12}$ ***miles.***

4. If six equivalent unit fractions are added together and the sum is one, what are the addends? Explain in terms of unit fractions why your answer makes sense.

Answer: $\frac{1}{6}$***; because adding 6 copies of*** $\frac{1}{6}$ ***results in 1.***