Building Concepts: What is an Exponent?

Lesson Overview

Algebraic Focus: Why are exponents useful and how do they behave?

An exponent indicates how many times the base to which it refers is a factor; i.e., in 3^4, the 4 indicates that the base, 3, is used as a factor four times. This lesson focuses on how the definition can be used in a variety of situations to make sense of the computations: a^b, $(a \cdot c)^b$, $(a^b)^c$ and in particular, allows students to investigate whether exponents "distribute" over operations other than multiplication.

Any sequence of multiplications may be calculated in any order and the numbers may be grouped together any way.

Learning Goals

1. Use and interpret the definition of an exponent;
2. write a numerical multiplication expression using whole number exponents;
3. create equivalent numerical multiplication expressions using whole number exponents.

Prerequisite Knowledge

What is an Exponent? is the first lesson in a series of lessons that explore the concept of expressions. Lessons in this series build on the knowledge from previous lessons. Prior to working on this first lesson, students should understand:

- the concept of multiplication of whole numbers;
- the commutative and associative properties of multiplication.

Vocabulary

- **exponent:** a small number written above and to the right of the base number, telling how many times the base number is used as a factor.
- **base:** the number being multiplied.
- **factor:** numbers that are multiplied together to produce another number.
- **cube:** to raise a number to the third power.
- **square:** to raise a number to the second power
- **power:** an expression that represents the repeated multiplication of the same factor.

Lesson Pacing

This lesson should take 50–90 minutes to complete with students, though you may choose to extend, as needed.

Lesson Materials

- Compatible TI Technologies:

  TI-Nspire CX Handhelds, TI-Nspire Apps for iPad®, TI-Nspire Software

- What is an Exponent_Student.pdf
- What is an Exponent_Student.doc
- What is an Exponent.tns
- What is an Exponent_Teacher Notes
- To download the TI-Nspire activity (TNS file) and Student Activity sheet, go to http://education.ti.com/go/buildingconcepts.

Class Instruction Key

The following question types are included throughout the lesson to assist you in guiding students in their exploration of the concept:

Class Discussion: Use these questions to help students communicate their understanding of the lesson. Encourage students to refer to the TNS activity as they explain their reasoning. Have students listen to your instructions. Look for student answers to reflect an understanding of the concept. Listen for opportunities to address understanding or misconceptions in student answers.

Student Activity: Have students break into small groups and work together to find answers to the student activity questions. Observe students as they work and guide them in addressing the learning goals of each lesson. Have students record their answers on their student activity sheet. Once students have finished, have groups discuss and/or present their findings. The student activity sheet can also be completed as a larger group activity, depending on the technology available in the classroom.

Deeper Dive: These questions are provided to facilitate a deeper understanding and exploration of the content. Encourage students to explain what they are doing and to share their reasoning.

Mathematical Background

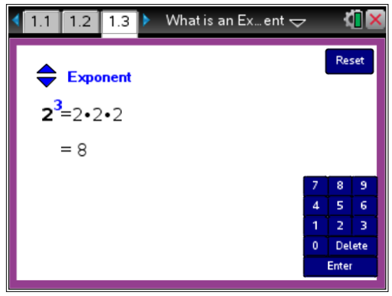
Exponents play a major role in mathematics as building blocks for understanding and performing the operation of multiplication in purely numerical situations and in algebraic situations involving variables. In Grade 4 students gain familiarity with factors and multiples. In Grade 5 they used whole number exponents as a compact way to express powers of 10. In Grade 6, students expand these notions to use exponents as a compact way to write expressions involving whole number multiplication. An exponent indicates how many times the base to which it refers is a factor; i.e., in 3^4, the 4 indicates that the base, 3, is used as a factor four times. This lesson focuses on how the definition can be used in a variety of situations to make sense of the computations: a^b, $(a \cdot c)^b$, $(a^b)^c$ and in particular, allows students to investigate whether exponents "distribute" over operations other than multiplication.

The *CCSS-M Progressions* document for Expressions and Equations refers to the "any order, any grouping" property, which in this case is a combination of the commutative and associative properties indicating that any sequence of multiplications may be calculated in any order and that the numbers may be grouped together any way. Using this notion of any order any grouping for multiplication along with "When in doubt, write it out", students explore equivalent ways of writing numerical expressions involving exponents. The lesson is not focused on developing "rules of exponents" or short cuts in procedures as students need to thoroughly understand the definition before they resort to formulas, and in fact, the formulas can be delayed until later in their study of mathematics because any exponent problem they are apt to encounter involving integers can be solved using "When in doubt, write it out" and the definition of an exponent.

A solid foundation in exponents is important because of the connection to exponential growth and decay, exponential functions in general and to logarithms as the inverse of exponential functions. Common misconceptions include not understanding which element in the expression is the base of the exponent, multiplying the exponent and the base, and reversing the role of the base and the exponent. It is important to note that this activity lays the foundation for work with rational exponents.

Part 1, Page 1.3

Focus: Students experience the ideas of exponent and base.



On page 1.3, use the arrows on the screen or on the keypad to change the exponent.

Moving the cursor over the base will display a pencil. Select the base to erase the number and then type on a new base using the keypad.

Base allows the base to be changed by typing a new base value.

Reset resets the page.

TI-Nspire Technology Tips

menu accesses the page options.

tab toggles between base and exponent.

ctrl del resets the page.

Teacher Tip: Note that the questions in the following section address some of the standard misconceptions students have about exponents, such as $2^3 = 6$.

Class Discussion

Have students…	Look for/Listen for…
Look at page 1.3. Make a conjecture about each of the following, and then check your conjecture using the file.	
• ***What is the value of 2^5? 2^1?***	Answer: 32 and 2
• ***Marko says that as the exponent increases, the value of the expression doubles. Saide says that you add two each time. What would you say to Marko and Saide?***	Answer: Marko is correct because each time you increase the exponent, you multiply by another factor of 2. Saide is not correct because you are not adding 2 but multiplying by 2; even though $2^1 = 2$ and $2^2 = 4$; and it looks like you are adding 2, $2^3 = 8$, $2^4 = 16$, and $2^5 = 32$ and you are not adding any twos after the first two times.

Class Discussion (continued)

- ***If you change the 2 to 4, what do you think 4^2 will be? 4^4 ?***

  Answer: 16 and 256

- ***Use the file to decide which of the following is true. Explain why in each case.***

  a. $9^3 = 27$ **b.** $3^9 = 27$

  c. $10^1 = 10$ **d.** $3^6 = 9^3$

  Answer: a) is not true because you use 9 as a factor 3 times which will give you 729. b) is not true because 3 is a factor 9 times and 27 only has 3 as a factor 3 times or $3 \cdot 3 \cdot 3$. c) is true because 10 used as a factor once is just 10. d) is true because they both equal 729. Some students may note that 9 is 3^2 and so 9^3 is the same as $(3^2)^3$, which when written out would be the product of six 3's $(3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3)$.

Student Activity Questions—Activity 1

1. Find at least three ways to obtain each of the following:

a. 64

Answer: 2^6, 4^3, 8^2

b. 531,441

Answer: 3^{12}, 9^6, 27^4

2. Which seems like the best definition of an exponent? Explain your reasoning.

An exponent __________.

a. is a multiplier

b. is a factor

c. tells how many times a number is used as a factor

d. tells you to multiply a number by another number

Answer: c. tells how many times a number is used as a factor.

3. Reset the page. Which of the following do you think is the base of the exponent in the expression 2^3 ? Explain your thinking.

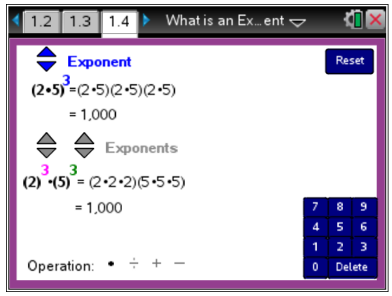
a. 2 **b. 3** **c. 2^3** **d. 8**

Answer: 2 is the base; it is the factor involved in the multiplication problem. We already know that 3 is the exponent, and 2^3 is the result.

> **Teacher Tip:** Be sure students agree with the answers to questions 2 and 3 so when those terms are used in later questions and discussions, students have the right mental image from which to reason.

Part 2, Page 1.4

Focus: Students use “when in doubt write it out” to investigate how a product raised to a power is related to the powers of individual factors and consider how the commutative and associative properties of multiplication can support their reasoning. They explore different sets of factors that produce the same result and consider whether an exponent can “distribute” over any operation.



On page 1.4, the arrows on the screen or on the handheld keypad to change the exponents.

Base chooses which base to be changed by typing a new base value.

Operation changes the operation being investigated.

TI-Nspire Technology Tips

menu accesses the page options.

tab toggles between bases and exponents.

×, ÷, +, – chooses the operation.

ctrl del resets the page.

Class Discussion

Have students…	Look for/Listen for…
Observe the two expressions on page 1.4	
• ***What is the difference between the expressions involving exponents on page 1.3 and the first expression on page 1.4?***	Answer: The expression on page 1.3 only had one number as the base; the first expression on page 1.4 has the exponent applying to a quantity marked by parentheses that contains two numbers multiplied together.
• ***How does the file show “when in doubt, write it out”?***	Answer: It shows three groups of the product of 2 and 5 and three groups of 2 and three groups of 5.
• ***Explain why it is reasonable that the two expressions have the same value.***	Answer: As long as the factors in two expressions are the same, the order does not make any difference and the products will have the same value.

Class Discussion (continued)

Make a conjecture about what you think will happen in each case. Then check your conjecture using the file.

- ***If you change all of the exponents in both expressions to 5.***

Answer: Two more sets of $(2 \cdot 5)$ will show up in the top expression and two more 2's in the set of 2's and two more 5's in the set of 5's in the bottom expression.

- ***If you change the exponent for the base 2 in the second expression to 4.***

Answer: The value of the second expression is multiplied by 32 because there will be five 4's instead of five 2's, so there are five more 2's as factors, which is 32.

Reset the page. Properties of operations can justify why some expressions have the same value.

- ***What properties of multiplication can be used to justify why the two expressions have the same value?***

Answer: The associative and commutative properties of multiplication justify why the expressions have the same value; changing the order and the grouping in a multiplication problem will not change the answer.

- ***Give an example to support your reasoning for the question above.***

Answers will vary: One example might be $3(2 \cdot 5) = (3 \cdot 2)5 = 30$.

An exponent of 2 is said to square the base, 3 is said to cube the base, and for whole number exponents greater than 3, the base is raised to that power- i.e., 2^6 would be 2 to the sixth power.

- ***Write 14 as two factors and cube the product. Is this the same as cubing each factor?***

Answer: The factors could be 1 and 14 or 2 and 7. Cubing the factors or cubing the product gives the same result 2,744.

- ***Use the file to raise 18 to the 7^{th} power. How did you find the result?***

Answer: Students might write 18 as the product of 3 and 6 or as the product of 2 and 9. 18 to the 7^{th} power is 612,220,032.

- ***Without using the file, decide whether the product of 2 to the 8^{th} power and 9 to the 6^{th} power will be less than 18 to the 7^{th} power. Explain your reasoning, then check using the file.***

Answer: Yes because 18 to the 7^{th} power is the same as the product of 2 to 7^{th} and 9 to the 7^{th}. The product of 2 to the 8^{th} and 9 to the 6^{th} will have one less 9 and one more 2, but one more 2 as a factor will not make up for the loss of a 9 as a factor.

Student Activity Questions—Activity 2

1. **Mari argued that $6^3 \cdot 2^3$ was the same as $3^3 \cdot 2^6$.**

   a. **Use the file to see if Mari is correct.**

   Answer: $6^3 \cdot 2^3 = 3^3 \cdot 2^6 = 1728$

   b. **Explain how "When in doubt, write it out" can help you see if she is correct.**

   Answer: $6^3 \cdot 2^3 = (6 \cdot 6 \cdot 6)(2 \cdot 2 \cdot 2) = ((2 \cdot 3)(2 \cdot 3)(2 \cdot 3))(2 \cdot 2 \cdot 2)$, which has six 2's as factors and three 3's as factors, the same as $3^3 \cdot 2^6$.

2. **Work with a partner to decide whether the following statements are true. Use the files if they will help your thinking. Explain why or why not in each case.**

   a. $\mathbf{5 \cdot 5^4 = 5^4}$

   b. $\mathbf{5 \cdot 5^4 = 25^4}$

   c. $\mathbf{5 \cdot 5^4 = 5^5}$

   d. $\mathbf{11^4 \cdot 11^6 = 11^{24}}$

   e. $\mathbf{11^4 \cdot 11^6 = 121^{10}}$

   f. $\mathbf{11^4 \cdot 11^6 = 121^{24}}$

   g. $\mathbf{11^4 \cdot 11^6 = 11^{10}}$

   h. $\mathbf{10^4 \cdot 11^6 = 110^{24}}$

   i. $\mathbf{10^4 \cdot 11^6 = 110^{10}}$

   Answer: c and g are true.

3. a. **Suppose the operation multiplication was replaced by the operation addition in both expressions. Do you think the top and bottom expressions will have the same value? Why or why not?**

   Answers will vary. Some may agree because it seems like addition and multiplication should behave the same way; others might use an example to show the two expressions will be different.

   b. **Change the operation to addition and check your answer to 3 a.**

   Answer: The values are different.

   c. **Try the operations of subtraction and division. Do either of these produce the same outcome for both expressions?**

   Answer: Subtraction produces different outcomes; division produces the same outcomes.

Student Activity Questions—Activity 2 (continued)

d. **Find a mathematical argument to help decide whether exponents "distribute" over the four operations.**

Answers will vary. One approach is to consider that exponents refer to factors and the only two operations that have anything to do with factors are multiplication and division (division can be thought of as reducing or dividing out common factors). Another approach is to remind students of strategies for multiplying two digit numbers $(25)(25)=5(25)+20(25)=5(20+5)+20(20+5)=5(20)+5(5)+20(20)+20(5)$ which may come through partial products or using an area model. These show that squaring a sum is not the same as the sum of the squares.

4. **Which of the following are true statements? Explain your reasoning in each case.**

a. **The product of two factors raised to a power is the same as the product of each factor raised to that power.**

Answer: True because you just get different orders and groups all connected by multiplication, so the result will be the same.

b. **The sum of two squared numbers is the same as the square of the sum of the numbers.**

Answer: False because exponents don't "distribute" over addition. You lose the partial products.

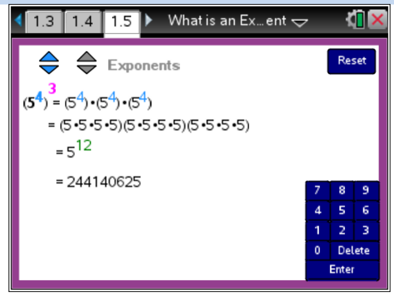
c. **The quotient of two numbers to a power can be thought of as the product of the numerator to the power and the power of the unit fraction corresponding to the denominator.**

Answer: True because $\left(\frac{a}{b}\right)^2=\frac{a}{b}\cdot\frac{a}{b}=\frac{a^2}{b^2}$.

d. **If you cube two numbers and then subtract, you will get the same answer as if you subtract the two numbers and then cube the answer.**

Answer: False. "Distributing exponents" does not work over subtraction.

Part 3, Page 1.5



Focus: Students investigate a base to a power that is raised to a power.

Page 1.5 functions in a similar way to page 1.4.

Class Discussion

Have students…

Look for/Listen for…

Look at the expression, $\left(5^4\right)^3$, on page 1.5.

- ***Tami says there are two bases in the expression. Do you agree or disagree with her? Explain your reasoning***.

  Answer: She is correct because the base of the exponent 3 is 5^4 and the other is 5, the base of the exponent 4.

- ***How does "When in doubt, write it out" seem to apply to the expression on page 1.5?***

  Answer: Three sets of 5^4 is rewritten as three sets of $(5\cdot5\cdot5\cdot5)$ to have twelve 5s in the multiplication problem.

- ***What do you think $\left(5^3\right)^4$ will be? Check your answer using the file.***

  Answer: The same as $\left(5^3\right)^4$ or 5^{12}

- ***Find at least two other ways to write an expression using exponents that will have the same value as $\left(5^3\right)^4$.***

  Answer: $\left(5^3\right)^4 = 244{,}140{,}625$. $\left(5^2\right)^6$ and $\left(5^2\right)^6$ will give the same result.

Student Activity Questions—Activity 3

1. **Explain the difference among: $(7^3)^2$, $(7^2)^3$, $(3^7)^2$, and $(3^2)^7$. Use the file to help your thinking.**

   Answer: $(7^3)^2 = (7^2)^3 = 117{,}649$. The base is always 7, in $(7^3)^2$, you have two sets of three 7's. In $(7^2)^3$ you have three sets of two 7's. In each case you have six 7's. The base in $(3^7)^2$ and $(3^2)^7$ is 3, and in each case you will have fourteen 3's.

2. **Do you agree or disagree with the following statements? Explain your thinking in each case.**

   a. **The number 5 has no exponent.**

   Answer: The number 5 has no explicitly written exponent, but it is understood that the problem has one factor of 5; i.e., $5^1 = 5$.

   b. **If you have five sets where each set has four 15s, you will have nine 15s.**

   Answer: No, you will have twenty 15's. Each set has four 15's, so for two sets, there would be eight 15's; three sets would have twelve; four sets would have sixteen; and five sets would have twenty 15's.

   c. **If a multiplication problem has two factors of 5 and two factors of 9, you could write the problem as two factors of 45**.

   Answer: Yes, because you can use the "any order any group" property to rearrange the factors to have two sets of 5 and 9 or two sets of 45.

Deeper Dive – Page 1.3

Have students…	Look for/Listen for…
One way to think about working with exponents is to remember the definition and think about how to expand the problem using the definition, i.e. "When in doubt, write it out". Identify which of the following pairs will be larger for each of the following. You can use the file to help your thinking, but give a reason for your answer that involves the definition of exponents.	
• 2^8 ***and*** 8^2	Answer: 8^2 has two 8's: $8 \cdot 8$ but each 8 has three 2's as factors so: $8^2 = 8 \cdot 8 = (2 \cdot 2 \cdot 2)(2 \cdot 2 \cdot 2)$. $2^8 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2$ or eight 2's as factors, which is more than six 2's so 2^8 is larger.

Deeper Dive – Page 1.3 (continued)

- ***10^2 and 2^{10}***

Answer: because

$10^2 = 10 \cdot 10 = 2 \cdot 5 \cdot 2 \cdot 5 = 2 \cdot 2 \cdot 5 \cdot 5$

$2^{10} = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2$

The product of two 5's is 25 and of only five 2's is 32, which is larger than 25. So 2^{10}, which has eight 2's as factors, is the larger.

Deeper Dive – Page 1.4

Identify the original factors in the file that produced each answer. Explain your thinking.

- ***The answer was 100,000,000.***

Answer: The answer has 8 zeros, so it is. This factors into $5^8 \cdot 2^8$.

- ***One of the factors was 5, and the answer was 1,728,000*.**

Answer: $(24 \cdot 5)^3$ and $24^3 \cdot 5^3$. Because the last three digits are 0's, there have to be three factors of 10, which means three factors of 5. So the exponent is 3. The other factor has at least three twos so it could be 8. Trial and error will give the factor as 24. (Another 2 or a three is the next logical choice; other numbers produce a product that is too large.). Some students might know the factoring rule that if the sum of the digits is divisible by 3, the number is divisible by 3, which gives 24 as a candidate for the missing factor.

Answer each. Explain your reasoning in each case.

- ***Which is easier to compute: $\frac{2 \cdot 2 \cdot 2}{6 \cdot 6 \cdot 6}$ or $\left(\frac{2}{6}\right)^3$?***

Answers will vary. It should be easier to reduce first to $\frac{1}{3}$ and then compute but students might disagree.

- ***Which is larger $\left(\frac{5}{3}\right)^2$ or $\frac{5^3}{3^4}$?***

Answer: $\left(\frac{5}{3}\right)^2$ is larger because the other expression is different by a factor of $\left(\frac{5}{9}\right)$ and multiplying by a fraction less than one will make the product smaller.

Sample Assessment Items

After completing the lesson, students should be able to answer the following types of questions. If students understand the concepts involved in the lesson, they should be able to answer the following questions without using the TNS activity.

1. Order the expressions from least to most:

   5^2 3^1 2^5 1^3

   Answer: $\boldsymbol{1^3, 3^1, 5^2, 2^5}$

2. Which is equivalent to 10000?

   a. $5^2 \cdot 2^5$ b. $5^5 \cdot 2^5$ c. $(5 \cdot 2)^4$ d. $5^5 \cdot 2^2$

   Answer: c. $\boldsymbol{(5 \cdot 2)^4}$

3. What is the value of $(3.4)(10^2)$

   a. 3.4 b. 34 c. 340 d. 3400

   Answer: c. 340

4. What is equivalent to $(4^3)^2$?

   a. 12^2 b. 16^3 c. 4^5 d. 8^3

   Answer: b. $\boldsymbol{16^3}$

5. Which equations with exponential expressions are true? Select all that apply.

   a. $3^3 = 3 \cdot 3$

   b. $5^2 = 5 \cdot 5$

   c. $5^4 = 4 \cdot 4 \cdot 4 \cdot 4 \cdot 4$

   d. $7 \cdot 7 \cdot 7 \cdot 7 \cdot 7 \cdot 7 = 6^7$

   e. $7 \cdot 7 \cdot 7 \cdot 7 \cdot 7 \cdot 7 = 7^6$

   f. $7 \cdot 7 \cdot 7 \cdot 7 \cdot 7 \cdot 7 = 7^7$

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Answer: b. $\boldsymbol{5^2 = 5 \cdot 5}$ ***and e.*** $\boldsymbol{7 \cdot 7 \cdot 7 \cdot 7 \cdot 7 \cdot 7 = 7^6}$

Building Concepts: What is an Exponent?

Student Activity Solutions

In these activities you will explore and write numerical multiplication expressions using whole number exponents. After completing the activities, discuss and/or present your findings to the rest of the class.

Activity 1 [Page 1.3]

1. Find at least three ways to obtain each of the following:

   a. 64

   Answer: 2^6, 4^3, 8^2

   b. 531,441

   Answer: 3^{12}, 9^6, 27^4

2. Which seems like the best definition of an exponent? Explain your reasoning.

   An exponent __________.

   a. is a multiplier

   b. is a factor

   c. tells how many times a number is used as a factor

   d. tells you to multiply a number by another number

   Answer: c) tells how many times a number is used as a factor.

3. Reset the page. Which of the following do you think is the base of the exponent in the expression 2^3? Explain your thinking.

   a. 2 b. 3 c. 2^3 d. 8

   Answer: 2 is the base; it is the factor involved in the multiplication problem. We already know that 3 is the exponent, and 2^3 is the result.

Activity 2 [Page 1.4]

1. Mari argued that $6^3 \cdot 2^3$ was the same as $3^3 \cdot 2^6$.

   a. Use the file to see if Mari is correct.

   Answer: $6^3 \cdot 2^3 = 3^3 \cdot 2^6 = 1728$

   b. Explain how "When in doubt, write it out" can help you see if she is correct.

   Answer: $6^3 \cdot 2^3 = (6 \cdot 6 \cdot 6)(2 \cdot 2 \cdot 2) = (2 \cdot 3)(2 \cdot 3)(2 \cdot 3))(2 \cdot 2 \cdot 2)$*, which has six 2's as factors and three 3's as factors, the same as* $3^3 \cdot 2^6$.

2. Work with a partner to decide whether the following statements are true. Use the files if they will help your thinking. Explain why or why not in each case.

   a. $5 \cdot 5^4 = 5^4$

   b. $5 \cdot 5^4 = 25^4$

   c. $5 \cdot 5^4 = 5^5$

   d. $11^4 \cdot 11^6 = 11^{24}$

   e. $11^4 \cdot 11^6 = 121^{10}$

   f. $11^4 \cdot 11^6 = 121^{24}$

   g. $11^4 \cdot 11^6 = 11^{10}$

   h. $10^4 \cdot 11^6 = 110^{24}$

   i. $10^4 \cdot 11^6 = 110^{10}$

   Answer: c and g are true.

3. a. Suppose the operation multiplication was replaced by the operation addition in both expressions. Do you think the top and bottom expressions have the same value? Why or why not?

   Answers will vary. Some may agree because it seems like addition and multiplication should behave the same way; others might use an example to show the two expressions will be different.

   b. Change the operation to addition and check your answer to 3a.

   Answer: The values are different.

   c. Try the operations of subtraction and division. Do either of these produce the same outcome for both expressions?

   Answer: Subtraction produces different outcomes; division produces the same outcomes.

   d. Find a mathematical argument to help decide whether exponents "distribute" over the four operations.

   Answers will vary. One approach is to consider that exponents refer to factors and the only two operations that have anything to do with factors are multiplication and division (division can be thought of as reducing or dividing out common factors). Another approach is to remind students of strategies for multiplying two digit numbers $(25)(25) = 5(25) + 20(25) = 5(20+5) + 20(20+5) = 5(20) + 5(5) + 20(20) + 20(5)$ *which may come through partial products or using an area model. These show that squaring a sum is not the same as the sum of the squares.*

4. Which of the following are true statements? Explain your reasoning in each case.

   a. The product of two factors raised to a power is the same as the product of each factor raised to that power.

      Answer: True because you just get different orders and groups all connected by multiplication, so the result will be the same.

   b. The sum of two squared numbers is the same as the square of the sum of the numbers.

      Answer: False because exponents don't "distribute" over addition. You lose the partial products.

   c. The quotient of two numbers to a power can be thought of as the product of the numerator to the power and the power of the unit fraction corresponding to the denominator.

      Answer: True because $\left(\frac{a}{b}\right)^2 = \frac{a}{b} \cdot \frac{a}{b} = \frac{a^2}{b^2}$.

   d. If you cube two numbers and then subtract, you will get the same answer as if you subtract the two numbers and then cube the answer.

      Answer: False. "Distributing exponents" does not work over subtraction.

Activity 3 [Page 1.5]

1. Explain the difference among: $(7^3)^2$, $(7^2)^3$, $(3^7)^2$, and $(3^2)^7$. Use the file to help your thinking.

   Answer: $(7^3)^2 = (7^2)^3 = 117{,}649$. *The base is always 7, in* $(7^3)^2$, *you have two sets of three 7's. In* $(7^2)^3$, *you have three sets of two 7's. In each case you have six 7's. The base in* $(3^7)^2$ *and* $(3^2)^7$ *is 3, and in each case you will have fourteen 3's.*

2. Do you agree or disagree with the following statements? Explain your thinking in each case.

   a. The number 5 has no exponent.

      Answer: The number 5 has no explicitly written exponent, but it is understood that the problem has one factor of 5; i.e., $5^1 = 5$.

   b. If you have five sets where each set has four 15's, you will have nine 15's.

      Answer: No, you will have twenty 15's. Each set has four 15's, so for two sets, there would be eight 15's; three sets would have twelve; four sets would have sixteen; and five sets would have twenty 15's.

   c. If a multiplication problem has two factors of 5 and two factors of 9, you could write the problem as two factors of 45.

      Answer: Yes, because you can use the "any order any group" property to rearrange the factors to have two sets of 5 and 9 or two sets of 45.