Building Concepts: Solving Systems of Equations Algebraically

TEACHER NOTES

Lesson Overview

In this TI-Nspire lesson, students will investigate pathways for solving systems of linear equations algebraically.

There are many effective solution pathways for a system of two linear equations. They are all similar in that they involve 4 main steps. 1) use the equations in two variables to create a new equation in only one variable, 2) solve the new equation, 3) substitute that variable's solution value into one of the original equations in the system of equations, and 4) solve that equation for the other variable.

Learning Goals

1. Understand that the initial key strategic goal in solving systems of linear equation in two variables is to produce a linear equation in one variable;
2. use algebraic techniques to solve a system of linear equations in two variables, in particular the elimination method and substitution;
3. determine efficient or elegant approaches to finding a solution to a system of linear equations in two variables
4. relate an algebraic solution to a system of equations in two variables to a graphical representation.

Prerequisite Knowledge

Solving Systems of Linear Equations is the seventeenth lesson in a series of lessons that explores the concepts of expressions and equations. In this lesson, students use the ideas developed in previous lessons to solve a system of linear equations in two variables. Prior to working on this lesson students should have completed *Solving Equations, Equations of the Form* $ax+b=c$ *Using Structure to Solve Equations,* and *What Is a Solution to a System of Linear Equations.*

Students should understand:

- that a solution to a system of linear equations corresponds to the point of intersection of the graphs of those linear equations; and
- how to use different approaches when solving a system of linear equations.

Vocabulary

- **system of linear equation in two variables:** comprises two or more equations where a common solution to the equations is sought
- **substitution:** the replacement of one value with another
- **solution to a system:** the point of intersection of the graphs of those linear equations

Lesson Pacing

This lesson should take 50–90 minutes to complete with students, though you may choose to extend, as needed.

Lesson Materials

- Compatible TI Technologies:

  TI-Nspire CX Handhelds, TI-Nspire Apps for iPad®, TI-Nspire Software

- Solving Systems of Equations Algebraically_Student.pdf
- Solving Systems of Equations Algebraically_Student.doc
- Solving Systems of Equations Algebraically.tns
- Solving Systems of Equations Algebraically_Teacher Notes
- To download the TI-Nspire activity (TNS file) and Student Activity sheet, go to http://education.ti.com/go/buildingconcepts.

Class Instruction Key

The following question types are included throughout the lesson to assist you in guiding students in their exploration of the concept:

Class Discussion: Use these questions to help students communicate their understanding of the lesson. Encourage students to refer to the TNS activity as they explain their reasoning. Have students listen to your instructions. Look for student answers to reflect an understanding of the concept. Listen for opportunities to address understanding or misconceptions in student answers.

Student Activity: Have students break into small groups and work together to find answers to the student activity questions. Observe students as they work and guide them in addressing the learning goals of each lesson. Have students record their answers on their student activity sheet. Once students have finished, have groups discuss and/or present their findings. The student activity sheet also can be completed as a larger group activity, depending on the technology available in the classroom.

Deeper Dive: These questions are provided for additional student practice and to facilitate a deeper understanding and exploration of the content. Encourage students to explain what they are doing and to share their reasoning.

Mathematical Background

This lesson consolidates student understanding of solving a system of linear equations in two variables using the ideas developed in several previous lessons; in particular, students learned that a solution to a system of linear equations corresponds to the point of intersection of the graphs of those linear equations. The goal in an algebraic approach to solving a system of linear equations is to transform the system into one equation in one variable in order to solve an equation of the form $ax+b=c$ or $ay+b=c$. It is important that students recognize that the solution to the system needs to specify values for both *x* and *y* corresponding to the point of intersection (x, y). The solution has to simultaneously make both equations true, *i.e.,* both true at the same time. For this reason, these systems are sometimes called *systems of simultaneous equations*. The equations in the systems in this lesson will require no initial collection or combining of terms.

If you assume that the same values of *x* and *y* make both equations true, then those same values will make the sum of the two equations true as well because "equals added to equals" produces another equality, (*i.e.,* $a=b$ and $c=d$ implies that $a+c=b+d$ since $a+c=b+c$ and $b+c=b+d$).

Typically, this is done algebraically using the addition/subtraction (elimination) method or the substitution method. In this lesson, students are encouraged to think about several different ways to approach finding the solution. Once they know different approaches can be used in solving a system of linear equations, students are asked to think about choosing a method that is most sense-making or efficient for a given system.

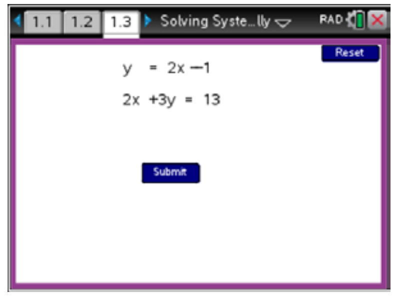
This lesson is grounded in research by (Maciejewski &Star, 2016) that suggests that having students deliberately compare strategies with focused questions to engender discussions of what they understand about different strategies for solving the same problem, how the strategies compare and how they connect results in better learning and retention.

Maciejewski, W., & Star, J.R. (2016). Developing flexible procedural knowledge in undergraduate calculus. *Journal of Mathematics Education.* DOI:10.1080/14794802.2016.1148626. To link to this article: http://dx.doi.org/10.1080

Building Concepts: Solving Systems of Equations Algebraically

Part 1, Page 1.3

Focus: There are many effective pathways for solving a system of two linear equations including: transforming an equation into an equivalent equation, adding two equations, substituting an expression from one equation in another equation, substituting a value for a variable in an equation.



On page 1.3, **Submit** shows a system (use **menu> Equations** for other systems). Select or **tab** highlight an equation.

Add Eq. adds two highlighted equations.

TI-Nspire Technology Tips

menu accesses page options.

tab cycles through the onscreen options.

ctrl del resets the page.

Transform allows operations on equations. **Expand** carries out the distributive property. **Collect** combines like terms. **Substitute** inserts a value or expression for an expression. Use **Submit Solution**, **Check It!** and then **Graph** to see results. **Reset** restarts with the current system.

Class Discussion

Have students…	**Look for/Listen for…**
Consider the first equation on page 1.3. Select* Submit *to set up the system of linear equations.	
• ***Select* Submit Solution *and enter values for x and y that make the first equation true (select the second coordinate of the ordered pair above Submit Solution to enter the value for y). Select* Check It!. *What do you observe?***	Answer: The ordered pair makes the first equation true, but in most cases will not make the second equation true.
• ***Reset. Enter values for x and y that make the second equation true. When you* Check It! *what do you observe?***	Answer: The ordered pair makes the second equation true, but in most cases will not make the first equation true.
• ***Recall that a solution to a system of equation has to make both equations true at the same time. Should you keep trying numbers until you find a solution to the system? Why or why not?***	Answer: Probably not because I could be lucky or I could guess forever.
Reset*.*	
• ***Make a conjecture about what the* Add Eq. *button does, then check your conjecture by selecting this button.***	Answer may vary: I guess the two equations will be added together to get $2x+4y=2x+12$.

Class Discussion (continued)

- ***Gill says that if x is 1, then y has to be 3. Do you agree? Why or why not?***

  Answer: $2+4y=2+12$ or $4y=12$, so the value of *y* must be 3.

- ***Sal says y has to be 3 no matter what value x has. Do you agree with Sal? Why or why not?***

  Answer: Because there is a 2*x* on both sides of the equation, the value of 2*x* will be the same on both sides no matter what the value of *x,* and to make the two sides equal 4*y* has to be equal to 12, ($4y=12$). So, $y=3$.

Substitute Var. ***allows you to substitute a value for a variable in an equation. To substitute 3 for y in the last equation, select*** **Substitute Var.*****, type 3 in the*** $y=$ ***blank and*** **Enter*****.***

- ***Now you have a new equation with only one variable (x). What values of x make this equation true?***

  Answer: Like Sal said, any value of *x* makes this true. (This equation is an identity.)

- ***To substitute 3 for y in each of the other two equations, select Substitute var., highlight the equation and type 3 for*** $y=$ ***again and Enter. Each of these equations has only one variable and you know how to solve these. Is there a single value of x that satisfies all of the new equations?***

  Answer: $x=2$ makes all of these equations true.

- ***Explain how you found your answer to the question above.***

  Answer will vary: Some students will explain how they used the highlight method or solution preserving moves to solve a series of equations. Other students might solve only one equation using those methods and check the value they found in the other one (that they had not solved).

- ***Select*** **Submit Solution** ***and enter the values you found for x and y as you did before. Select*** **Check It!** ***and*** **Graph*****. Interpret the displayed graph in terms of the algebraic solution.***

  Answer: If the solution is correct, the intersection point of the two lines is the solution. If the solution is incorrect, then the point will not be at the intersection of the two lines.

Reset***. Here is another way to begin to solve a system of equations. Select*** **Substitute Exp*****. and for Choose variable: enter y.***

Class Discussion (continued)

- ***What do you notice?***

Answers may vary: The variable y is colored orange in both equations, the expression $2x-1$ is highlighted in the first equation, and an arrow appears from the highlighted expression to the orange y in the second equation.

- ***Press* Enter. *What happens? What is displayed?***

Answer: The expression $2x-1$ for y from the first equation is substituted for y in the second equation. $2x+3(2x-1)=13$ is displayed.

- ***Expand and collect like terms, then solve the resulting equation. Is your solution the solution to the linear system of equations? Why or why not?***

Answer: The solution to this equation in one variable is $x=2$. A solution to a system of two equations is either a pair of numbers, $(x, y)=(\_\_\_,\_\_\_)$, No solution or Same line. So, $x=2$ is not a solution to the linear system of equations because it is only one number.

- ***What is the solution to the system of linear equations? Explain how you found your answer.***

Answer may vary: You substitute the value of x in one of the original equations and solve for y. The solution to the first equation, $y=2x-1$, is $y=3$, but the solution to the system of equations is the ordered pair $(x, y)=(2, 3)$.

Reset***. The system of equations can be solved in yet another way by adding two equations to eliminate one of the variables.***

- ***How can you transform the first equation so that, when you add the transformed equation to the second equation, one of the variables will be eliminated? Make a conjecture and then use the TNS activity to see if your conjecture is right. Explain your reasoning.***

Answer: Add $-2x$ to both sides of the second equation because the $-2x$ in one equation and the $2x$ in the other will add to 0, and the result will not have x as a variable.

- ***What is the resulting equation? What is its solution?***

Answer: The resulting equation is $4y=12$. The solution is $y=3$.

- ***You can substitute the value for y into one of the original equations to find the value for x. Which of the two equations should you use and why?***

Answers will vary. It does not make any difference which equation you use because the corresponding value for x will be the same so long as you don't make a mistake.

Class Discussion (continued)

• ***Find the solution to this system and explain your reasoning.***	Answer: Select Substitute var. to substitute the value 3 for *y* in one of the first two equations, solve the resulting equation in one variable for *x*, and submit the solution of the two equations in one variable as the ordered pair $(2, 3)$. Check that your solution makes both of the original equations true. (Note this is the same solution as above)
Vivian claims that while the steps were different, the strategies for solving the system in the previous problems are very similar: 1) use the equations in two variable to create a new equation in only one variable, 2) solve the new equation, 3) substitute that variable's solution value into one of the original equations in the system of equations, and 4) solve that equation for the other variable. Do you agree? Why are why not?	Answer: Vivian is correct in that this strategy leads to finding values for each variable. However, neither of those variable values by itself is the solution, so it is important to give the pair (x, y) as the solution. Note: This strategy is one you can use on all systems of linear equations in two variables.

Student Activity Questions—Activity 1

1. **Hil was solving $-2x + y = -1$ and $2x + 3y = 13$ by adding the two equations to find the value for *y*. She wondered which of the following steps she might take to find the value for *x*.**

   a. **substitute the value she found for y into $-2x + y = -1$.**

   b. **substitute the value she found for y into $2x + 3y = 13$.**

   c. **multiply the first equation by -3 and add the result to $2x + 3y = 13$.**

   d. **multiply the first equation by -3 and the second equation by -1 and add the two equations.**

   e. **substitute the value she found for y into both of the original equations**.

   Answer: Any of the choices would work to find the value for *x* because they would all result in one equation with the variable *x*.

Part 2, Page 1.3

Focus: Students compare solution pathways for solving systems of linear equations with the goal of identifying those that are strategic, efficient and elegant.

Class Discussion

A flowchart representing different pathways for solving the system of linear equations in Part 1, $\begin{cases} y = 2x - 1 \\ 2x + 3y = 13 \end{cases}$ ***is given below.***

Have students…

- ***Describe the strategies in each of the pathways.***

Look for/Listen for…

Answers will vary: The pathway on the left could be described as: Because I can add two equations together and collect like terms I formed a new equation. Because I can add $-2x$ to the equation, I can get an equation in one variable. Because I can use the cover-up method, I can solve this equation in one variable for *y*. Because I can substitute the value of *y* in both the original equations, I can get two equations in *x*. Using the cover-up method on either of those equations, I can find the value of *x* (the other variable). The solution would be the ordered pair solution $(2, 3)$. Descriptions of the other paths will be similar.

Class Discussion (continued)

- ***How are the pathways alike? How are they different? Could you always use any of the approaches to solve a system? Why or why not?***

  Answers will vary. The two pathways on the right involve substitution, while the one on the left is just adding the two equations and then the numbers had to make sense because you had $\_ + 4y = \_ + 12$ and so you only had to think about the 4*y* and the 12. This will only work if the numbers in the equations are just right. The second and third pathways can be used only if you have some common term in both of the equations. The pathways substituted into different equations to find the value of *x*, but they all gave the same value for *y*.

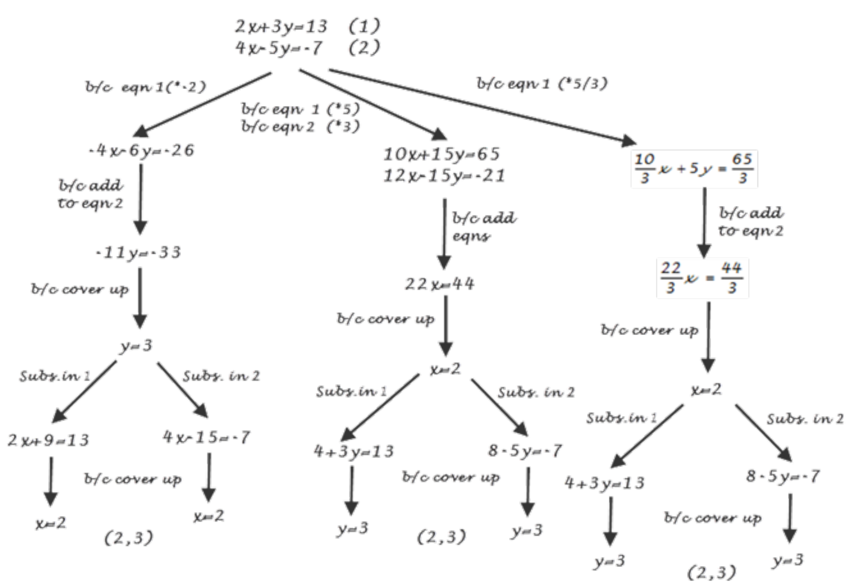
- ***Which of the pathways seems most efficient to you? Why? Which pathway would be easiest to use? Why?***

  Answers will vary. Some students might point out that the system is set up to use the substitution method so that would be the most efficient and the easiest to use. If both of the equations were like the second equation, the substitution method might not be so easy because there would be no "$y =$" or "$x =$" to directly substitute.

- ***Mathematicians talk about "elegant" solutions – where elegant means insightful or clever; often involving a relatively short way or immediate approach to a problem. Did any of the pathways seem elegant to you? Explain your reasoning.***

  Answers will vary: The solution on the left could be elegant because you really just have to add the two equations to see how to find the solution. The one on the right seems to be obvious because the equation is set up to use substitution.

The teacher created a chart of different pathways that could be used to solve the system of linear equations $\begin{cases} \mathbf{2x + 3y = 13} \\ \mathbf{4x - 5y = -7} \end{cases}$.



Class Discussion (continued)

Have students…	Look for/Listen for…
• ***Explain how each pathway works.***	Answers will vary: For the solution on the left: 1) Because I can multiply the first equation by -2, I obtain a new equation that when added to the second equation will result in an equation that will only have one variable because the $4x$ and $-4x$ will add to 0. 2) Using the cover-up method, I can solve this equation for *y*. 3) Because I can substitute this value in either or both of the original equations, I can obtain two equations in *x*. 4) Using the cover-up method again, I can solve either equation for *x*, 5) I know the solution is $(2, 3)$ because it makes both of the original equations true. The other descriptions would be similar.
• ***Compare the pathways. What is the difference between the approaches in the pathways? What is similar about the approaches? Which of the pathways seems most efficient to you? Why?***	Answers will vary: The two pathways on the left are alike because they both involve adding the two equations to get the multipliers of *x* as opposites. The one on the right is different because it uses the substitution method. After I find a value for one of the variables, the solutions look the same. I think the solution on the left is most efficient because I only have to multiply one equation before I add equations.
• ***Which pathway seems easiest? Would you use this approach on every problem? Why or why not? Which one seems the most difficult? Why?***	Answers will vary. The pathway on the left seems the easiest because you only have to multiply one equation to get the multipliers to be opposites. This approach won't work every time because sometimes the numbers might be "unfriendly"--like 5 and 7 (prime). If one of the equations is set up like *y*= or *x*=, it would probably be easier to use the substitution method. The pathway on the right is harder and would probably lead to more mistakes than the others because you have to use fractions.

Class Discussion (continued)

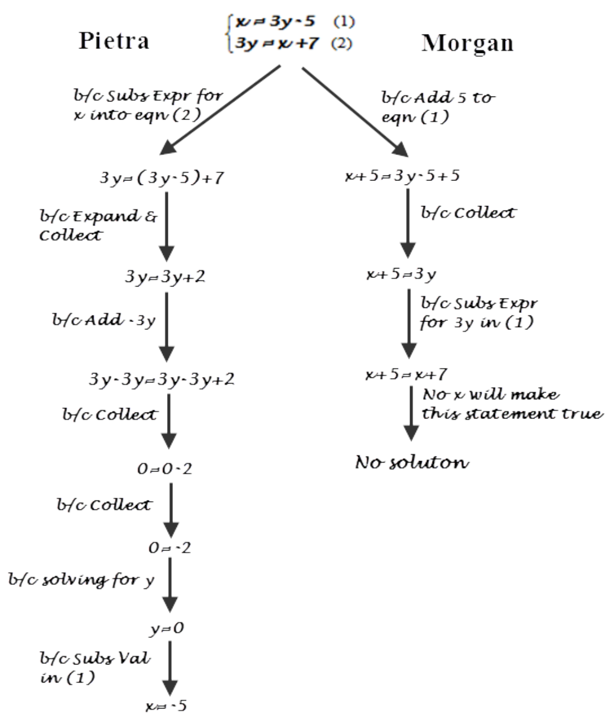
Use* menu> Equations *to choose Set 4.

- ***Which, if any, of the strategies you saw above could you use to solve the system of linear equations: $\begin{cases} x = 3y - 5 \\ 3y = x + 7 \end{cases}$?***

  Explain your reasoning.

  Answers will vary: I substituted the expression for *x* from the first equation, $3y - 5$, in the second equation to obtain an equation in one variable, *y*.

- ***Pietra and Morgan solved the system using each of the two pathways below. How are their approaches different?***



Answer: They both used the substitution method, but Morgan substituted for *x* and Pietra for *y.*

- ***Did they get the same or different answers? Explain how you know whether they are both correct.***

  Answer: They got different answers: Morgan said there is no solution, and Pietra gave an order pair $(5, 0)$ for the solution. Checking the values in the system shows $5 \neq 3(0) - 5$ and $3(0) \neq 5 + 7$. The ordered pair does not make either of the equations true. The graphs show the two lines representing the equations are parallel and will not intersect, so there is no solution. Morgan is correct.

Class Discussion (continued)

- ***How can analyzing the slope of the two equations in the system help you understand who is correct? Is just looking at the slope enough to tell? Why or why not?***

Answer may vary. If two equations have the same slope, they either represent the same line or parallel lines. You need a point to tell you which situation you have. For example, in this system, the point (1, 2) works in equation 1 but not 2, although the slopes of both lines are $\frac{1}{3}$. Because they have the same slope but the same point does not make them both true statements, the lines are parallel.

Student Activity Questions—Activity 2

1. **A solution to a system of equations in two variables is an ordered pair of values that specify what each variable has to be to make both equations true. Sometimes the system actually represents the same constraints on the variables, with the same slope for the two equations and the same points make both equations true and so the solution is represented by all of the points on line. Sometimes the two equations have the same slope but the same points do not make both of them true; they represent parallel lines and the system does not have a solution.**

   a. **If you find a value for *x* that satisfies an equation, have you solved the system? Why or why not?**

   Answer: No, the solution is either an ordered pair--two values--or no values or all of the points on a line.

   b. **If you found the value for x in a system of equations, a logical strategy is to substitute that value into one of the original equations to find the value for *y*. Is this always a good path to take? Give an example to support your reasoning**.

   Answer: This will always work, but if the value for *x* is a fraction and the equations are not directly stated as a function of *y* or of *x*, it might be better to “start over” and find the value for y. For example, in $2x+5y=17$ and $3x-4y=12$, once the value for x has been found to be $\frac{128}{23}$, it would be much easier to “start over” and multiply each equation to eliminate the *y* values when the equations are added.

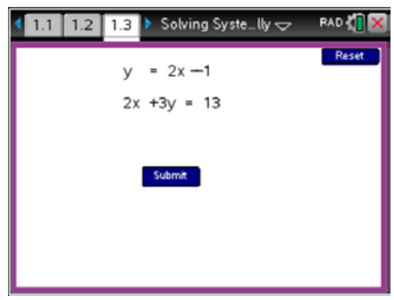
Student Activity Questions—Activity 2 (continued)

c. **In solving one system of equations, Sol got a statement that said $2=2$ and in another he got the statement $2=0$. What is the difference in these two statements and what do they indicate about the solutions for the system?**

Answer: The first statement is true and could be likened to $x+2=x+2$, a statement that is true for all values of *x*. This indicates that the solution for the system is an infinite set of ordered pairs, i.e., all of the points on the line represented by either of the equations (as the two equations are equivalent, they have the same solution set.) The statement $2=0$ is false, and tracing it back to the original equations indicates that no number will make those statements both true at the same time.

Part 3, Page 1.3

Focus: Students gain experience with systems of equations that have different structures and helping them make sensible choices of solution strategies given those structures (flexible procedural knowledge).



Student Activity Questions—Activity 3

1. **Reset. Use *menu*> *Equations*. Solve each of the systems in Sets 2, 3,5, and 6 in two ways: by the addition method to eliminate a variable and by substitution. Then answer the questions below.**

    a. **Which of the two methods did you prefer for this system? Why?**

    b. **Would you use this method on every system of equations? Why or why not?**

    c. **How do you know if your solutions are correct?**

    Answers will vary. Be sure to students have the opportunity to discuss their answers to the three questions above with each other and as a class. It is the discussion as well as their own work that will help them make smart choices for strategies. Note that checking by substitution into the original equations is the only way you can tell if you have found the right solution if the solution is an ordered pair. Graphing or finding the slope and testing a point in both equations will tell you if the solution set is all of the points on a line or if the system has no solution.

Student Activity Questions—Activity 3 (continued)

Set 2

$\begin{cases} 2x+3y=13 \\ 4x-5y=-7 \end{cases}$; solution (2, 3)

Set 3

$\begin{cases} 2x+3y=13 \\ 4x-5y=-7 \end{cases}$; solution (2, 3)

Set 5

$\begin{cases} 3y=x+7 \\ 2x-6y=-14 \end{cases}$; solution "Same Line"

Set 6

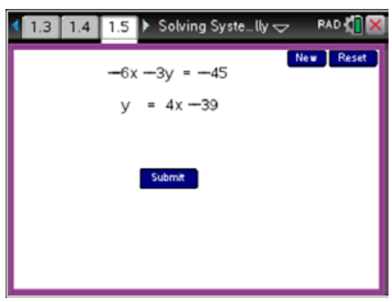
$\begin{cases} y=2x-4 \\ y=4x-10 \end{cases}$; solution (3, 2)

Part 4, Page 1.5

Focus: Use solution preserving moves to find unknown values of the shapes for a mobile when some of the shapes have fixed values.

On page 1.5, **New** generates a new system of equations that can be solved using the TNS activity.

Reset returns to the most recent system.



TI-Nspire Technology Tips

menu accesses page options.

tab cycles through the variables and constants in each equation.

ctrl del resets the page.

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TEACHER NOTES

Class Discussion

Have students…	Look for/Listen for…
Use* New *to generate two equations they would like to solve using substitution, two equations they would like to solve adding two equations and using the elimination method, and two equations that they can solve by thinking about an "elegant" approach. Have classmates solve each other's equations and see if they agree with each other's thinking.	Answers will vary. Examples of substitution: 1) $-y = -x + 1$ and $-4x = 7y + 81$; 2) $-5y = 3x + 5$ and $x = -3y - 3$ Elimination: 1) $-3y = -8x - 19$ and $-2x + 5y = 9$; 2) $5y = 2x + 28$ and $-3x = -8y + 26$ "Elegant" approaches: 1) $7y = 4x - 71$ and $2y = -4x + 46$ (add); 2) $-3y = -9y + 6$ and $2x - y = 2$ (multiply 1 by $\frac{-1}{3}$ then add)

Deeper Dive

• ***Given the system, $x = 3y + 25$ and $6y - x = -43$, is adding the two equations a good strategy? Why or why not?***	Answers may vary. Adding produces the equation $6y = 3y - 18$ that does not have an *x*, so yes.; multiplying by $\frac{1}{3}$ gives $2y = y - 6$, so $y = 2$. Using that value in equation 1, $x = 31$.
• ***To solve the system $-7x + 4y = -58$ and $8x + 6y = 21$, Petra suggests multiplying the first equation by -3 and the second by 2. Morgan says you have to multiply the first equation by -6 and the second by 4 (or the first by 6 and the second by -4). Who is right and why?***	Answer: They are both correct. Each strategy will result in new equations where the multipliers of *y* are opposites and so adding the two equations will eliminate the *y*-variable and produce an equation in only *x*.
• ***Sami found this system of equations in his sister's high school math book.*** $x + y + z = 12$ $x = 2z - 4$ $y = z - 8$ ***Explain how Sami could use one of the strategies for solving a system of two equations in two variables to solve this system.***	Answers will vary. He might substitute the expressions for x and y from the last two equations into the first equation to get $2z - 4 + z - 8 + z = 12$ so the result is one equation with one variable; $4z = 24$ and $z = 6$. Substituting the value for z into the last two equations, $x = 8$ and $y = -2$.

Sample Assessment Items

After completing the lesson, students should be able to answer the following types of questions. If students understand the concepts involved in the lesson, they should be able to answer the following questions without using the TNS activity.

1. A system of equation is shown.

$$\begin{cases} x = 10 \\ 3x + 5y = 20 \end{cases}$$

What is the solution (x, y) of the system of equations?

PARCC Practice Test 2016

***Answer:* $(10, -2)$**

2. A chemist has two acid solutions. Solution *A* contains 10% acid, and solution *B* contains 30% acid. He will mix the two solutions to make 10 liters of a third solution, solution *C*, containing 25% acid.

The system of equations shown can be used to represent this situation.

$$\begin{cases} x + y = 10 \\ 0.10x + 0.30y = 2.5 \end{cases}$$

What is the number of liters of solution *B* the chemist mixes with solution *A* to create solution *C* containing 25% acid?

Adapted from PARCC Sample Test Question 2015

***Answer:* $(2.5, 7.5)$**

3. Joe solved this linear system correctly.

$$\begin{cases} 6x + 3y = 6 \\ y = -2x + 2 \end{cases}$$

These are the last two steps of his work.

$6x - 6x + 6 = 6$

$6 = 6$

Which statement about this linear system must be true?

a. x must equal 6

b. y must equal 6

c. There is no solution to this system.

d. There are infinitely many solutions to this system.

Adapted from Smarter Balance 2014

Answer: D

4. At Jorge's local video store, "new Release" video rentals cost $2.50 each and "Movie Classic" video rentals cost $1.00 each (including tax). On Saturday evening, Jorge rented 5 videos and spent a total or $8.00.

   How many of the 5 rentals were New Releases and how many were Movie Classics?

   New Releases ____________ Movie Classics ____________

   NAEP 2015

   Answer: New Releases 2 and Classics 3

5. What is the solution to the system of equations $\begin{cases} 3x - 2y = -7 \\ x + y = 11 \end{cases}$?

   Answer: $x =$ ______________________ $y =$ ______________________

   NAEP 2009

   Answer: $\boldsymbol{x = 3; y = 8}$

6. Mary is buying tickets for a movie.

   - Each adult ticket costs $9.
   - Each child ticket costs $5.
   - Mary spends $110 on tickets.
   - Mary buys 14 total tickets.

   What are the total number of adult tickets and total number of child tickets she buys?

   Adapted from the 2016 California Assessment of Student Performance and Progress Practice Test.

   Answer: 10 adult tickets and 4 child tickets.

Student Activity Solutions

In these activities, you will investigate pathways for solving systems of linear equations algebraically.

Activity 1 [Page 1.3]

1. Hil was solving $-2x + y = -1$ and $2x + 3y = 13$ by adding the two equations to find the value for *y*. She wondered which of the following steps she might take to find the value for *x*.

   a. substitute the value she found for y into $-2x + y = -1$.

   b. substitute the value she found for y into $2x + 3y = 13$.

   c. multiply the first equation by -3 and add the result to $2x + 3y = 13$.

   d. multiply the first equation by -3 and the second equation by -1 and add the two equations.

   e. substitute the value she found for y into both of the original equations.

   Answer: Any of the choices would work to find the value for x because they would all result in one equation with the variable x.

Activity 2 [Page 1.3]

1. A solution to a system of equations in two variables is an ordered pair of values that specify what each variable has to be to make both equations true. Sometimes the system actually represents the same constraints on the variables, with the same slope for the two equations and the same points make both equations true and so the solution is represented by all of the points on line. Sometimes the two equations have the same slope but the same points do not make both of them true; they represent parallel lines and the system does not have a solution.

   a. If you find a value for *x* that satisfies an equation, have you solved the system? Why or why not?

      Answer: No, the solution is either an ordered pair--two values--or no values or all of the points on a line.

   b. If you found the value for x in a system of equations, a logical strategy is to substitute that value into one of the original equations to find the value for *y*. Is this always a good path to take? Give an example to support your reasoning.

      Answer: This will always work, but if the value for x is a fraction and the equations are not directly stated as a function of y or of x, it might be better to "start over" and find the value for y. For example, in $2x + 5y = 17$ *and* $3x - 4y = 12$*, once the value for x has been found to be* $\frac{128}{23}$*, it would be much easier to "start over" and multiply each equation to eliminate the y values when the equations are added*.

c. In solving one system of equations, Sol got a statement that said $2 = 2$ and in another he got the statement $2 = 0$. What is the difference in these two statements and what do they indicate about the solutions for the system?

Answer: The first statement is true and could be likened to $x + 2 = x + 2$*, a statement that is true for all values of x. This indicates that the solution for the system is an infinite set of ordered pairs, i.e., all of the points on the line represented by either of the equations (as the two equations are equivalent, they have the same solution set.) The statement* $2 = 0$ *is false, and tracing it back to the original equations indicates that no number will make those statements both true at the same time.*

Activity 3 [Page 1.3]

1. Reset. Use **menu> Equations**. Solve each of the systems in Sets 2, 3,5, and 6 in two ways: by the addition method to eliminate a variable and by substitution. Then answer the questions below.

   a. Which of the two methods did you prefer for this system? Why?

   b. Would you use this method on every system of equations? Why or why not?

   c. How do you know if your solutions are correct?

   Answers will vary. Be sure to students have the opportunity to discuss their answers to the three questions above with each other and as a class. It is the discussion as well as their own work that will help them make smart choices for strategies. Note that checking by substitution into the original equations is the only way you can tell if you have found the right solution if the solution is an ordered pair. Graphing or finding the slope and testing a point in both equations will tell you if the solution set is all of the points on a line or if the system has no solution.

   Set 2

   $\begin{cases} 2x + 3y = 13 \\ 4x - 5y = -7 \end{cases}$; solution (2, 3)

   Set 3

   $\begin{cases} 2x + 3y = 13 \\ 4x - 5y = -7 \end{cases}$; solution (2, 3)

   Set 5

   $\begin{cases} 3y = x + 7 \\ 2x - 6y = -14 \end{cases}$; solution "Same Line"

   Set 6

   $\begin{cases} y = 2x - 4 \\ y = 4x - 10 \end{cases}$; solution (3, 2)