Building Concepts: Linear Inequalities in One Variable

TEACHER NOTES

Lesson Overview

In this TI-Nspire lesson, students focus on inequalities and methods of solving them, building on the concept of solution- and order-preserving moves in the context of an inequality.

Finding the solution to an inequality in one variable involves four ideas:

1. the smaller of two number is to the left of the larger;
2. if one number is less than another, $a < b$, then some positive *c* added to *a* will equal *b* $(a + c = b)$;
3. the point of equality (boundary point) divides a set of values into those greater than and those less than that point; and
4. some operations on both sides of an inequality preserve order and some do not (in particular, multiplying or dividing by a non-zero number).

Learning Goals

1. Recognize that the solution for a linear inequality in one variable is an unbounded interval with an infinite set of values or will not exist;
2. identify order-preserving moves for inequalities;
3. recognize the difference between finding the solution for a linear inequality in one variable and finding the solution for a linear equation in one variable;
4. solve and justify the solution for a linear inequality in one variable.

Prerequisite Knowledge

Linear Inequalities in One Variable is the eighth lesson in a series of lessons that explores the concepts of expressions and equations. In this lesson, students solve inequalities by building on the concept of solution- and order-preserving moves. This lesson builds on the concepts of previous lessons. Prior to working on this lesson, students should have completed *Equations and Operations*, *Using Structure to Solve Linear Equations,* and *Using Structure to Solve Equations*. Students should understand:

- how arithmetic operations applied to both sides of an inequality affect the order of the values of the resulting expressions;
- how to perform operations with negative numbers;
- how to locate negative numbers on a number line.

Vocabulary

- **inequality:** a relationship between two expressions that are not equal.
- **variable:** a letter or symbol that stands for a changing quantity.
- **equation:** a statement that represents the equality of two expressions.
- **solution-preserving moves:** performing the same operation to both sides of an equation to produce a new equation with the same solution.

Lesson Pacing

This lesson may take several days to complete with students.

Lesson Materials

- Compatible TI Technologies:

  TI-Nspire CX Handhelds, TI-Nspire Apps for iPad®, TI-Nspire Software

- Linear Inequalities in One Variable_Student.pdf
- Linear Inequalities in One Variable_Student.doc
- Linear Inequalities in One Variable.tns
- Linear Inequalities in One Variable_Teacher Notes
- To download the TI-Nspire activity (TNS file) and Student Activity sheet, go to http://education.ti.com/go/buildingconcepts.

Class Instruction Key

The following question types are included throughout the lesson to assist you in guiding students in their exploration of the concept:

Class Discussion: Use these questions to help students communicate their understanding of the lesson. Encourage students to refer to the TNS activity as they explain their reasoning. Have students listen to your instructions. Look for student answers to reflect an understanding of the concept. Listen for opportunities to address understanding or misconceptions in student answers.

Student Activity: Have students break into small groups and work together to find answers to the student activity questions. Observe students as they work and guide them in addressing the learning goals of each lesson. Have students record their answers on their student activity sheet. Once students have finished, have groups discuss and/or present their findings. The student activity sheet also can be completed as a larger group activity, depending on the technology available in the classroom.

Deeper Dive: These questions are provided for additional student practice and to facilitate a deeper understanding and exploration of the content. Encourage students to explain what they are doing and to share their reasoning.

Mathematical Background

The systematic study of equations began with the *Equations and Operations*, which developed solution-preserving moves for equality; *Using Structure to Solve Linear Equations*, which developed the highlight method for analyzing equations; and *Visualizing Equations Using Mobiles*, which used solution-preserving moves on equations with variables on both sides of the equation to find the solution for the equation. This lesson focuses on inequalities and methods of solving them, building on the concept of solution- and order-preserving moves in the context of an inequality. An inequality makes a statement about the order of the values of two expressions. Adding, subtracting, multiplying, or dividing both expressions by the same number (non-zero number for multiplication or division) results in a new inequality statement. Understanding how arithmetic operations applied to both sides of an inequality affect the order of the values of the resulting expressions is essential to finding the solution set of an inequality involving a variable. Student confusion typically arises because multiplying both sides of an inequality by a negative value reverses the direction of the inequality.

In earlier grades, students learned that the largest value in a set of numbers is the one farthest to the right on the number line. They also learned notation for greater than and for less than, although interpreting the notation often is problematic for students as they continue in mathematics and one reason why careful attention should be paid to ways of writing and reading inequality statements. By definition, the number *a* is less than the number *b*, $a < b$, if there is a positive number *c* such that $a + c = b$. Another important concept is the *trichotomy property of real numbers* which states that for any two numbers, *a* and *b*, exactly one of the following is true: $a > b$, $a < b$, or $a = b$. This idea leads to the notion that the point of equality is the boundary point for determining whether the solution set for a linear equation in one variable will be either to the left or right of that point. Selecting a point and checking the value of the expression in the inequality can locate the solution.

The lesson has four parts: 1. interpreting simple linear inequalities with one variable and developing the meaning of a solution set for an inequality; 2. solution- and order-preserving moves for inequalities; 3. solving simple linear inequalities; and 4. compound inequalities. To be sure students have time to explore and develop their understanding of the concepts, the lesson may take several days. Students should be familiar with locating negative numbers on the number line. The lesson is written for students with experience with operations with negative numbers. Parts 1 and 4 can be used with all students. For level one students (those who have not yet studied operations with negative numbers), versions of Part 2 and Part 3 involving only positive rational numbers are located after the Assessment section of the lesson.

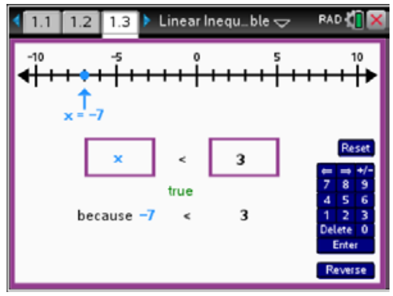
An additional resource for inequalities can be found at https://www.teachingchannel.org/videos/teaching-inequalities.

Part 1, Page 1.3

Focus: The solution set to a linear inequality is a set of numbers that make the inequality statement true.

On page 1.3, students can use the arrow keys or drag the point to move it. They can change the number in the inequality by selecting the box with the constant term, typing a new value, and pressing **enter**. They can change the inequality by selecting the inequality.

Reverse changes the inequality from $a < b$ to $b < a$.



TI-Nspire Technology Tips

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Class Discussion

Given any two numbers, one is either greater than, less than, or equal to the other.

- ***How do you know that –7 is less than 3?*** Answers will vary: One answer might be that -7 is to the left of 3.
- -7 ***is how much less than 3?*** Answer: 10 units.
- ***Move the point on page 1.3. Describe what happens to the dot as you move the point to the right and left.*** Answer: For numbers less than 3, the dot is solid. For numbers 3 and more than 3, the dot is not solid.
- ***What do the words "true" and "false" tell you?*** Answer: When the *x*-value is less than 3, the word "true" appears because the statement is true; when the *x*-value is more than 3 or equal to 3, the word "false" appears because the statement is false.

Change the number on the right side of the inequality to –1.

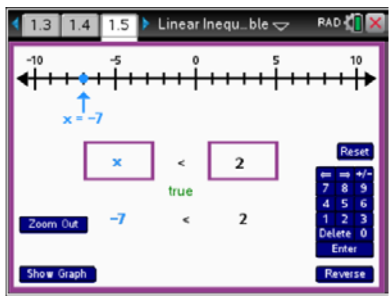
- ***Identify three values for*** x ***for which the word "true" appears. Explain what "true" means in this context.*** Answers will vary: -2, -3, -4. Whenever "true" appears, the value of *x* is a number that is to the left of -1.

Class Discussion (continued)

- ***Move x to -7. What number added to -7 will equal -1?*** — Answer: 6.
- ***Move the dot. Describe the collection of numbers that seem to make the statement true.*** — Answers may vary. Some may say whole numbers to the left of -1; others may say all numbers to the left of -1.
- ***Select Reverse. How is the new statement in the box related to the original statement?*** — Answer: The statement went from $x < -1$ to $-1 > x$, but the original and new statements communicate the same message.

Part 1, Page 1.5

Focus: The solution set to a linear inequality is the set of all numbers that make the inequality statement true.



Page 1.5 functions in a similar way to page 1.3. When the box with the constant term is highlighted, students can type in a new value for the number and press **enter**. When the box with the linear expression is highlighted, they can type in a value to be added or subtracted from *x*.

Show Graph displays the solution set on the number line.

Zoom Out extends the number line.

Zoom In retracts the number line.

TI-Nspire Technology Tips

menu accesses page options.

tab cycles between the left and right boxes and the buttons.

enter selects the constant term or activates reverse.

ctrl del resets the page.

Class Discussion

Have students… | **Look for/Listen for…**

Consider the inequality on page 1.5.

- ***Are -7, -1, or 5 solutions to the inequality? Explain why or why not.*** — Answer: -7 and -1 are solutions because they are to the left of 2 so they are both smaller than 2, and the statements $-7 < 2$ and $-1 < 2$ are both true. 5 is to the right of 2 and so $5 < 2$ is a false statement.

Class Discussion (continued)

• ***To find the solution set for an inequality means collecting all of the values that make the inequality true. What is the set of solutions for the inequality on page 1.5?***	Answer: All of the numbers less than 2.
• ***Select*** **Show Graph*****. Explain how this represents the collection of all of the values that make the inequality true.***	Answer: The shaded area stops at $x = 2$, showing that all numbers including fractions right up to 2 but not including 2 are elements of the solution set. The shaded area for the solution does not continue beyond 2 because none of the numbers to the right of 2 are less than 2.
• ***Select*** **Zoom Out*****. How does this change the graph?***	Answer: The number line now represents values of *x* from -100 to 100. The graph shows that all of the numbers to the left of 2 are less than 2 and elements of the solution set.
Select **Zoom In*****. Change the inequality sign to an equal sign by selecting the inequality sign several times.***	
• ***Move the dot. Where is the dot solid? Explain why.***	Answer: The dot is solid at when *x* is 2 because 2 is the solution to the equation, the value that makes the statement (the equation $x = 2$) true.
• ***How is the solution for a linear inequality different from the solution for a linear equation?***	Answer: The solution for an inequality is a whole set of numbers satisfying the statement; the solution for an equation is one number unless there is no solution or the equation is an identity.
• ***Describe the numbers on the graph in relation to the number 2.***	Answer: All of the numbers to the left of 2 are less than 2; the numbers to the right of 2 are more than 2.
Reset. ***Select*** **Show Graph*****. Change the inequality to each of the following and describe how the solution set changes each time and how you can tell from the graph.***	
• **>**	Answer: The solution is all of the numbers to the right of 2, not including 2. The solution set is shaded with an empty circle at 2.

Class Discussion (continued)

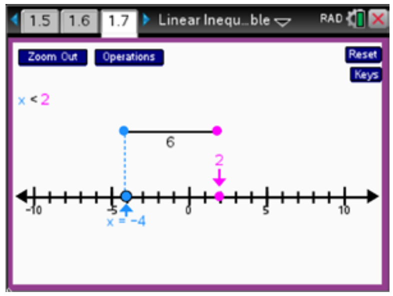
• $\geq$	Answer: The solution is all of the numbers to the right of 2 and including 2. The solution set is shaded with a solid circle at 2.
• $\neq$	Answer: All numbers but 2. The solution set is everything shaded except an empty circle at 2.

Reset.

• ***Create the inequality $-3 > x$. Describe how you created the inequality.***	Answers may vary. One approach might be to change the 2 to -3 and select **Reverse**. Another might be to select **Reverse** and then change the 2 to a -3.
• ***Describe the solution set for $-3 > x$. What is the boundary for the solution set?***	Answer: The solution set is all of the numbers to the left of -3. The boundary for the solutions set is the number -3.
• ***Move* x *to 8. What number added to 8 will equal -3? Explain how you found your answer.***	Answer: $-11+8=-3$. Explanations will vary. Some students may count on the graph; others might think about a missing addend.
• ***Is $-\frac{11}{3}$ a member of the solution set? Why or why not?***	Answer: Yes. Reasons may vary: $-\frac{11}{3}$ is to the left of -3; since $-\frac{11}{3}+\frac{2}{3}=-3$, a positive number added to $-\frac{11}{3}$ makes -3 which means that $-\frac{11}{3}$ is less than -3.
When you learned how to solve equations, you learned some solution-preserving moves, that is, you learned moves that could be used to create a new equation that would have the same solutions as the original equation. Make a conjecture about order-preserving moves you think will create inequalities that will have the same solutions as the original inequality.	Answers will vary. Students may think you can add, subtract, multiply, or divide both sides of the inequality and retain the solution.

Part 2, Page 1.7

Focus: Develop a sense of "order- and solution-preserving" moves that can be applied to inequalities and how these moves relate to the solution set for an inequality.



On page 1.7, students use tab to cycle between the pink and blue dots and **Zoom Out/Zoom In**. To move the dots, students grab and drag or **tab** to highlight the dot and use the right/left arrow keys. To add an operation, select **Operations** or **menu> Operations**. To change the constant that has been added, multiplied, etc., students tap or select **enter** to display an *x* in the lower left corner of the screen.

Keys shows the keypad to type in a value.

Selecting an **Operation** creates an inequality using one of the four operations.

TI-Nspire Technology Tips

[menu] accesses page options.

[tab] cycles between pink and blue dots.

[enter] selects the constant term or activates reverse.

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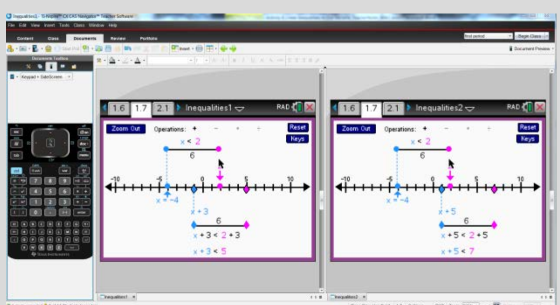
Class Discussion

Teacher Tip: Working with a partner provides opportunities for students to compare solution sets for different inequalities. The teacher also can demonstrate two solution sets by creating a copy of the TNS activity with a new name, then using the multiple screen option on the software to show both screens at the same time. (Activate the multiple screen option by right clicking on the second activity at the bottom of the screen and selecting new vertical group.)



Discussion questions and student activity questions for Part Two included here are designed for students prepared to multiply and divide with negative numbers. For questions that include only positive numbers, see the section after the Assessment Questions.

Class Discussion (continued)

These questions revisit the idea that if $a < b$, then a positive number c exists to make $a + c = b$.

Have students…	Look for/Listen for…
Go to page 1.7. Work with a partner.	
• ***What values are associated with the two dots on the number line?***	Answer: The blue dot is at *x* which is -4 and the pink dot is at 2.
• ***What does the 6 represent on the line segment?***	Answer: The 6 indicates the length of the segment which is the distance between -4 and 2.
Select Operations and the plus sign.	
• ***What values are associated with the two diamonds on the number line?***	Answer: The blue diamond is at $x+5$ which is 1, and the pink diamond is at $2+5$ which is 7.
• ***What does the 6 related to the new segment represent?***	Answer: The 6 indicates the length of the segment or the distance between between 1 and 7.
• ***Remember if $a < b$ (here $x < 2$), then some positive number c will make $a + c = b$. Using the graph, what is a positive number you can add to -4 to get to 2? to $(-4)+5$ to get to 7?*** ***In both cases, write the algebraic sentence that supports your answer.***	Answers: In both cases, you would add 6: $-4+6=2$ and $(-4+5)+6=7$.
• ***How are the two inequalities on the screen related?***	Answer: If you add 5 to both sides of $x < 2$, you will get $x+5<7$.
Move the pink dot to 4 and the blue dot (x) to 1.	
• ***Give the two inequalities shown on the screen and describe how they are related.***	Answer: $x < 4$ and $x+5<9$ are the two inequalities. If you add 5 to both sides of the first inequality, you get the second.
• ***What do the 3s represent?***	Answer: The 3s are the length of the two segments and for $x=1$, represent the distance between *x* and 4 and between $x+5$ and $4+5$ or 9.
• ***Find a positive number, c, so that when $x=1$, $x+c=4$ and $(x+5)+c=9$.***	Answer: For $1+c=4$, the equation would be $1+3=4$, and for $(1+5)+c=9$, the equation would be $(1+5)+3=9$.

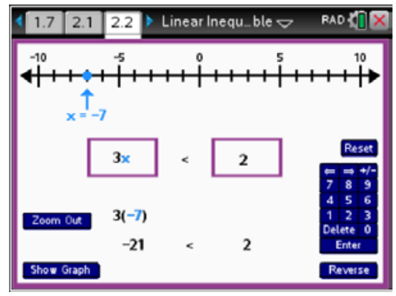
Class Discussion (continued)

- ***Change the added value to 2. How would the question and answer for the question above change?***

Answer: The question would be: Find a positive number, *c*, so that when $x = 1$, $x + c = 4$ and $(x+2)+c = 6$. The answer for $1+c = 4$ would be $1+3 = 4$ and for $(1+2)+c = 6$ would be $(1+2)+3 = 6$.

Part 2, Page 2.2

Focus: Develop a sense of "order- and solution-preserving" moves that can be applied to inequalities and how these moves relate to the solution set for an inequality.



On page 2.2, students use tab to cycle through the three numbers and the buttons; to change the numbers, select the coefficient, addend, or constant and type new values.

The inequality can be changed by selecting or by **menu> Inequality**.

menu> Constant changes the number on the right side of the inequality.

Class Discussion

These questions investigate the effect of adding a number to both sides of an inequality and multiplying both sides of an inequality by a non-zero number. To investigate order-preserving moves in the situation $a < b$, students find the positive number *c* that when added to *a* will equal *b*.

Have students…	Look for/Listen for…
Go to page 2.2. Create $x < 3$ by selecting the space before the coefficient 3 in the inequality $3x < 2$, deleting the 3, and then selecting the right side of the inequality and entering 3. Your partner should create $x + 5 < 8$. Note: selecting the space to the right of x allows addition or subtraction of a number. Select **Show Graph**.	
• ***How do the graphs of the two inequalities compare? What does this tell you about the solution sets for the two inequalities?***	Answer: The graphs are the same, and so the solution sets will be the same, $x < 3$.
• ***Create $x < -2$ and $x + 4 < 2$. Compare the solution sets for the two inequalities.***	Answer: They are the same, $x < -2$.

Class Discussion (continued)

- ***Suppose $x+4<2$, if $x=6$, can you find a positive number to add to $(x+4)$ to get 2? Explain why or why not.***

  Answer: For $x=6$, no positive number added to $(6+4)$ would equal 2, because for $(6+4)+\text{number}=2$, the number would be -8; $10+-8=2$. So 6 is not a solution to $x+4<2$.

- ***Suppose $x+4<2$, if $x=-5$, can you find a positive number to add to $(x+4)$ to get 2? Explain why or why not.***

  Answer: For $x=-5$, the number would be 3 since $(-5+4)+3=-1+3=2$. There is a positive number that can be added to $(-5+4)$ to equal 2, -5 is a solution to $x+4<2$.

- ***Make a conjecture about how the solution sets will change when you add a number to both sides of an inequality. With your partner, create two examples to support your conjecture.***

  Answers may vary. Students should conclude that the solution sets are preserved.

Return to page 1.7 and **Reset**. ***With your partner, create the inequalities $x>-4$ and $x+3>-1$.***

- ***How do these inequalities differ from those in the earlier questions?***

  Answer: The inequalities are written as greater than rather than as less than.

- ***Locate x at 6. What would you add to -4 to get $x=6$? What would you add to -1 to get $x+3=6+3=9$?***

  Answer: $-4+10=6$, and $-1+10=9$.

- ***On page 2.2, create the two inequalities and compare the solution sets.***

  Answer: The solution sets are the same, $x>-4$.

- ***Create several more pairs of inequalities that are stated as greater than and compare the solution sets. Make a conjecture about how the solution set for inequalities that are "greater than" will change when you add the same value to both sides of the inequality.***

  Answer: The solution sets do not change.

Consider each of the following:

- ***Carrie says for any x that is less than 2, you can find a positive number to add to $(x+5)$ that will give you 7. Do you agree with Carrie? Why or why not?***

  Answer: She is right. She is thinking about the equation $(x+5)+c=7$ where $x<2$. So, *x* could be any number such as 1.5 or 1, or 0 or -3. The sum of one of those numbers and 5 will be less than 7, so some positive number *c* will be difference between the sum and 7.

Class Discussion (continued)

- ***Jose claimed you could add any number you want to both sides of an inequality and the direction of the equality would not change. Create an example that supports Jose's claim.***

  Answers will vary. One example might be $x > 4$ and $x + 3 > 7$. The solution set remains the same and the direction of the inequality remains "greater than."

On page 1.7, select the multiplication option.

- ***What number would you add to -3 to get to 2? What number would you add to $(-3)(3)$ to get to 6? Write number sentences that support your answers.***

  Answer: The numbers would be 5 and 15 respectively since $-3+5=2$ and $(-3)(3)+15=6$.

- ***How are these numbers different than the numbers you found in the adding problems?***

  Answer: In the adding problem, the numbers were the same in both cases. Here one number (and segment length) is three times as long as the other one.

- ***What might explain the difference? Make a conjecture and then check it out using a multiplier of 2.***

  Answer: When we added, it was just kind of a shift of everything up (or down if you subtracted or added a negative number). Now we are multiplying, and it seems reasonable that the differences would be multiplied as well.

> **Teacher Tip:** Note that the answer above brings in the idea of *transformations*: adding is a translation and multiplication is a dilation. These are not ideas students need to formalize at this time, but they do lay the foundations for connections when they study transformations later.

- ***If you multiplied by $\frac{1}{3}$, what do you think will happen to the segments? Use the division option and divide by 3 to check your conjecture. Explain why this is a reasonable way to check.***

  Answer: A typical conjecture would be that the second segment would be $\frac{1}{3}$ the length of the first; which is supported by the TNS activity where for $x=-3$, the distance between *x* and 2 or -3 and 2 is 5, but the distance between $-\frac{x}{3}$ and 2 or $-\frac{3}{3}=-1$ and 2 is $\frac{5}{3}$.

In the following questions, students encounter the fact that multiplying by a negative number does not preserve order because no positive number *c* exists that will make $a+c=b$. The key idea students should gain is that if the order of the inequality is to be preserved, some positive number *c* can be added to the smaller value to equal the larger value. What happens with multiplication by a negative number?

Class Discussion (continued)

Return to page 2.2.

- ***With your partner, create the inequalities $x < 3$ and $2x < 6$ and* Show Graph*.***

Answers may vary. Typically, students would think both inequalities have the same solution set.

- ***For each inequality, give an example of one number that is in the solution set and one number that is not in the solution set. Show why your examples are correct.***

Answers will vary. 0 is in the solution set for both; $0 < 3$ and $2(0) = 0 < 6$ because 0 is the left of both 3 and 6 on the number line. 8 is not in the solution set for either because 8 is to the right of both 3 and 6 on the number line.

Test	$0 < 3$	$2x < 6$
0	In: $0 < 3$	In: $2(0) = 0 < 6$
8	Not in: $8 > 3$	Not in: $2(8) = 16 > 6$

- ***Move the dot to $x = -5$. Explain how you know the solution set is all of the numbers less than 3 in both cases by finding the positive number that added to* x *will equal 3 and the positive number that added to 2*x *will equal 6. (You may want to use page 1.7 of the TNS activity.)***

Answer: $-5 + 8 = 3$, so 8 is the number for the first inequality; 16 is the number for the second inequality because $2(-5) + 16 = -10 + 16 = 6$.

- ***On page 1.7, create the inequalities $x < 3$ and $2x < 6$ and set $x = -5$. Explain what the two segments mean and why the pink diamond is to the right of the blue diamond.***

Answer: The segments represent the numbers you would add to the values for *x* on the left side of the inequality to equal the right sides (8 and 16). The pink diamond is to the right of the blue diamond because 2*x* for $x = -5$, 2*x* or -10, and 6 is greater than -10.

On page 2.2, with your partner, create the inequalities $x < 3$ and $-2x < -6$ and* Show Graph*.

- ***Is the solution set what you would expect? Why or why not?***

Answers may vary. Typically, students would think both inequalities have the same solution set.

Class Discussion (continued)

- ***For each inequality, give an example of one number that is in the solution set and one number that is not in the solution set. Show why your examples are correct.***

Answers will vary. 0 is in the solution set for $x<3$; $0<3$ because 0 is the left of 3 on the number line. However, 0 is not in the solution set for $-2x<-6$ because $-2(0)=0$ which is greater than 6 and 0 is to the right of -6 on the number line.

Test	$x<3$	$-2x<-6$
0	True: $0<3$	False: $-2(0)=0>6$
6	False: $6>3$	True: $2(6)=-12<-6$

- ***On page 1.7, multiply both sides of $x<3$ by -2. What happened to the order of the pink and blue diamonds? Explain why.***

Answer: The order changed; the pink diamond is now at -6 and the blue diamond at 10 because $-2(-5)=10$.

- ***For $x=-5$, what positive number added to $-2x$ will equal -6? Explain your reasoning.***

Answer: The only number that added to $-2(-5)$ to equal -6 is -16 because $-2(-5)-16=-6$. If you added 16, $-2(-5)+16=26$. To keep the less than, you have to be able to add a positive number to the smaller value to equal the larger value.

Student Activity Questions—Activity 1

1. What would you say to each of the following students?

a. Milo: "You can subtract any number from both sides of an inequality, and the solution set for the new inequality will be the same as the solution set for the original inequality."

Answers will vary. Milo is correct because adding a negative number is the same as subtracting, so the direction of the inequality will not change and the solution set will remain the same.

b. Mai: "Multiplying both sides of an inequality by a negative number reversed the order of the inequality sign."

Answers may vary. Mai is right, because for the inequality $x<3$, multiplying both sides by -2 yields $-2x>-6$. If $x=5$, you can think about what number added to -6 will equal $-2(-5)=10$-, and the number will be 16; $-6+16=(-2)(-5)$ Since we added a positive number to -6 so that it was equal to $-2(-5)$ or 10, switching the order (direction) of the inequality sign makes a true statement.

Student Activity Questions—Activity 1 (continued)

c. **Wu: "If you divide both sides of an inequality by the same number, the solution set for the new inequality will be the same as the solution set for the original inequality."**

Answers may vary: Wu is not correct because if you divide by a negative number the direction of the inequality reverses. Also you cannot divide by zero.

d. **Petra: "Because the product of two negatives is a positive, when you multiply or divide a set of negative numbers by a negative number, the resulting numbers are 'flipped' to the other side of the number line."**

Answers will vary. Petra is correct although they will not land the same "distance" from 0 unless the multiplier is -1.

2. **Use pages 1.7 and 2.2 of the TNS activity to help decide whether each of the following statements is true if $x < -7$. Describe what operation you could do on both sides of the inequality $x < -7$ to keep the solution set of the new inequality the same as the original solution set.**

a. $x + 3 < -4$ **b.** $3x < -21$ **c.** $x + 3 < -4x - 2 < -9$

d. $-3x > 21$ **e.** $-3x < 21$ **f.** $-7 > x$

Answer: a) adding 3 to both sides of the inequality will keep the solution set as $x < -7$; b) multiplying both sides by 3 will keep the solution set the same, $x < -7$; c) taking 2 away from both sides will keep the solution set the same, $x < -7$; d) multiplying both sides by -3 will reverse the direction of the inequality and keep the same solution set; e) not true, multiplying both sides of the inequality will change the solution set to $x > -7$; f) this is the same inequality as $x < -7$, just reversed.

Part 3, Page 2.2

Focus: Develop strategies for solving linear inequalities in one variable

The focus in the discussion questions is on finding the solution for a linear inequality in one variable using solution- and operation-preserving moves for inequalities. The first question introduces the notion of the point of equality as the boundary between greater than and less than relationships. In the following two sets of questions, students apply solution- and order-preserving moves to find solutions; the fourth set of questions returns to using the boundary point and testing a point as a way to find the direction of the inequality that is the solution set. The subsequent question reviews strategies for finding solutions to linear inequalities in one variable. The final question introduces the notion of eliminating terms with a negative multiplier for the variable by adding the opposite of the term to both sides and rewriting the inequality with a positive multiplier for the variable, eliminating the need to reverse the order for the solution set when multiplying both sides by a negative value.

Class Discussion

Discussion questions and student activity questions for Part Three included here are designed for students prepared to multiply and divide with negative numbers. For questions that include only positive numbers, see the section at the end of the Teacher Notes.

Have students…	Look for/Listen for…
Reset *page 2.2 and select* Show Graph*.*	
• ***Drag the dot. Does the shading stop at 0 or at 1? Why or why not?***	Answer: Neither, because a fraction like $\frac{1}{3}$ is larger than 0 but $3\left(\frac{1}{3}\right)=1<2$.
• ***At what value of* x *will the expression on the left be equal to 2? Explain how you found your answer.***	Answer: Thinking, what number times 3 will equal 2, gives the point of equality as $\frac{2}{3}$.
• ***Describe the solution set for the inequality* $3x<2$*. How do you know you have all the possible solutions?***	Answer: The solution set is the set of all of the numbers less than $\frac{2}{3}$. You have them all because using any number greater than $\frac{2}{3}$ for x will produce a false statement.
• ***Select* Zoom Out*. How does the graph support your answer to the question above?***	Answer: The graph shows all of the numbers less than $\frac{2}{3}$ are the solution, and those numbers keep going to the left of $\frac{2}{3}$.

Previously, students were asked how to change an inequality with just *x* on one side into a new inequality where a number was added to or multiplied by *x*. The following questions ask students to start from a more complicated inequality and find the solution set.

Create each of the inequalities by selecting the multiplier of* x*. Identify the solution- and order-preserving moves you could use to find the solution, then use the TNS activity to check your thinking.	
• $4x \geq -28$	Answer: multiply both sides by $\frac{1}{4}$; solution is $x \geq -7$.
• $-4x \geq 28$	Answer: multiply both sides by $-\frac{1}{4}$; solution is $x \leq -7$.

Class Discussion (continued)

- **$-4x \geq -28$**

Answer: multiply both sides by $-\frac{1}{4}$; solution is $x \leq 7$.

- **$28 < -4x$**

Answer: multiply both sides by $-\frac{1}{4}$; solution is $-7 > x$.

Choose the multiplier of x, then select to the right of x in the block. Subtract 4. Create each of the inequalities. Use the TNS activity to find the solution then identify a value in the solution set and show why it is in the solution set.

- **$x - 4 > -7$**

Answer: $x > -3$. Examples will vary: -1 because $-1 - 4 = -5 > -7$.

- **$7 > x - 4$**

Answer: $x < 11$. Examples will vary: 0 because $7 > 0 - 4 = -4$.

- **$-7 > x - 4$**

Answer: $x < -3$. Examples will vary: -5 because $-7 > 5 - 4 = -9$.

Find the point of equality for the expressions on each side of the inequality (the boundary point) by using what you know about the highlight method for solving equations. Then find the solution set for each inequality. Test points to determine the direction of the inequality. Check your solution using the TNS activity.

- **$-x - 4 > 2$**

Answer: The answer to the question "what minus 4 equals 2?" The answer would be 6, which implies that $-x = 6$, so $x = -6$. Checking a point on either side of the boundary, -6:

$x = -6$	Test value	$x - 4$	Is it >2?
> -6	0	$-(0) - 4 = -4$	False $-4 < 2$
< -6	-7	$-(7) - 4 = 7 - 4 = 3$	True $3 > 2$

The inequality was greater than 2, so the solution is $x < -6$.

Class Discussion (continued)

- $\mathbf{2x - 4 \geq 5}$

Answer: What minus 4 equals 5 implies that $2x = 9$, and what is multiplied by 2 to get 9 implies $x = \frac{9}{2}$.

$x = \frac{9}{2}$	Test value	$2x - 4$	Is it >5?
$> \frac{9}{2}$	5	$2(5) - 4 = 6$	True $6 > 5$
$< \frac{9}{2}$	0	$2(0) - 4 = -4$	False $-4 < 5$

The inequality was greater than or equal to 5, so the solution is $x \geq \frac{9}{2}$.

- $\mathbf{-2x + 4 > -5}$

Answer: What minus 4 equals -5 implies that $-2x = -1$, and what do you multiply by -2 to get -1 implies $x = \frac{1}{2}$.

$x = \frac{1}{2}$	Test value	$-2x - 4$	Is it >5?
$> \frac{1}{2}$	2	$-2(2) - 4 = -4 - 4 = -8$	False $-8 < -5$
$< \frac{1}{2}$	0	$-2(0) - 4 = 0 - 4 = -4$	True $-4 > -5$

The inequality was greater than -5 so the solution is $x < \frac{1}{2}$.

Student Activity Questions—Activity 2

1. Consider the inequality $7 - 2x \geq -5$. Find the solution set by

a. finding the point of equality and testing a point on either side to determine the order of the inequality describing the solution.

Answer: Thinking 7 minus what makes -5 means the missing addend is -12. What times -2 makes -12 means the missing factor is 6. Testing points:

$x = 6$	Test value	$7 - 2x$	Is it > -5?
> 6	8	$7 - 2(8) = 7 - 16 = -9$	False $-9 < -5$
< 6	0	$7 - 2(0) = 7 - 0 = 7$	True $7 > -5$

The inequality was greater than or equal so the solution is $x \leq 6$.

b. using a solution- and/or order-preserving move on both sides of the inequality.

Answer: Add a -7 to both sides to produce $-2x \geq -12$. Multiply both sides by $-\frac{1}{2}$ or divide by -2 to produce $x \leq 6$, remembering to change the order of the inequality.

2. Latashia suggested a new strategy.

a. She added 2*x* to both sides of the inequality. What was her result? Would the solution or order change? Why or why not?

Answer: $7 \geq -5 + 2x$; the order would not change nor would the solution, because you can add or subtract the same quantity to both sides of an inequality and still preserve the original solution set.

b. What do you think her next step could be?

Answers may vary. She could add 5 to both sides to get $12 \geq 2x$, then divide both sides by 2 to get $6 \geq x$.

c. What is an advantage of Latashia's method?

Answer: The multiplier of *x* is not negative, so you do not have to worry about changing the direction of the inequality, i.e., preserving order.

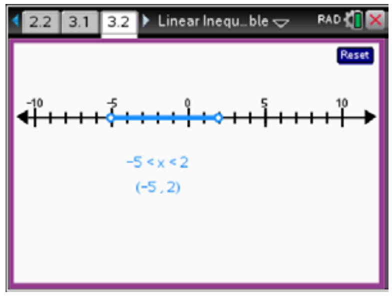
Part 4, Page 3.2

Focus: Writing and interpreting compound inequalities.

On page 3.2, students drag a dot or tab to select a dot, then use the arrow keys to move it.

Selecting the inequality symbol will cycle through the options; **menu> Inequalities** provides the same choices.

Selecting the boundary symbol cycles through the options; **menu> Boundaries** gives the same choices.



TI-Nspire Technology Tips

menu accesses page options.

tab toggles between the boundaries.

Arrow keys move left and right.

ctrl del resets the page.

Class Discussion

Have students…	Look for/Listen for…
Consider the graph on page 3.2.	
• ***Describe the shaded section of the number line represented in the graph.***	Answer: The graph shows all of the numbers between but not including -5 and 2.
• ***Describe the section of the number line that is not shaded.***	Answer: The non-shaded section consists of the numbers that are less than or equal to -5 and greater than or equal to 2.
• ***The notation below the graph might be familiar from earlier work with histograms. If -5 is to be included in the interval, how will the notation change? What will happen to the dot?***	Answer: The dot would be filled in and the notation would be $[-5, 2)$.
• ***Select the inequality sign and change it to include -5. Was your answer to the question above correct?***	Answers will vary depending on the answer to the question above.
Graph $[-2, 4)$. Explain how the graph of the following is different from the graph for $[-2, 4)$. **a.** $(-2, 4)$ **b.** $(-2, 4]$ **c.** $[-2, 4]$	Answer: The graph of $[-2, 4)$ includes -2 but not 4; a) includes neither end point; b) includes the 4 but not the -2; c) includes both endpoints.

Class Discussion (continued)

Which interval is the longest? Use the TNS activity to explain your reasoning.

a. $-3 < x < 3$ **b.** $-4 < x < 2$ **c.** $-3 < x < 2$

Answer: The intervals in a) and b) have the same length, 6 units. The interval in c) is 5 units long.

Tina downloaded more songs than Ashli but less than Tim. Tina downloaded at least three songs, which was less than the eight songs Tim downloaded. If x is the number of songs Tina downloaded, which of the following describes the number of songs she downloaded? (The TNS activity might help your thinking.) Explain your reasoning.

a. $3 > x > 8$ **b.** $3 < x < 8$

c. $-3 \le x < 8$ **d.** $-3 < x \le 8$

Answer: c) because at least 3 means the number of songs could be 3 or more but the number was less than 8, so it would be any number from 3 up to but not including 8.

Deeper Dive — Page 2.2

Think about equations and inequalities:

- ***If you add two inequalities with the same inequality sign, will the solution sets be the union of the two solution sets? Explain why or why not.***

  Answer: The solution set will not be the union of the two solution sets: consider $x < 3$ and $x < 5$: the solution would be $2x < 8$, which is $x < 4$. But the first solution set is the set of numbers less than 3, which does not include up to 4 (like $3\frac{1}{2}$), and the second solution set is all numbers less than 5, which is more than the numbers less than 4.

- ***If you add two equations, will the solution set be the union of the two solution sets?***

  Answer: Consider $2x = 10$ and $3x = 6$; the solution for the first is 5 and for the second is 2; the solution for $5x = 16$ is a fraction, $\frac{16}{5}$.

Deeper Dive — Page 1.3

- ***Are there some types of equations you can add, where the solution of the sum will be the same as the union of the solutions?***

  Answer: Equations that have the same solution. For example, $2x = 10$ and $3x = 15$ have a sum $5x = 25$. All the equations have the solution $x = 5$, so the union of the solutions is $x = 5$.

Deeper Dive — Page 1.3 (continued)

Use some subset of the numbers –9, –3, –1, 3, 4, and 12 to create four inequalities that have the same solution sets. Share your inequalities with a partner to see if you are thinking correctly.

Answers will vary. One possible set:
a) $-3x \le -9$; b) $4x \ge 12$; c) $-x \le -3$; d) $-3 \ge -x$.

All of these inequalities will have the same solution set, $x \ge 3$. When you multiply both sides by $-\frac{1}{3}$ in a) the inequality sign reverses and you have $x \ge 3$; multiplying both sides of b) by $\frac{1}{4}$ gives $x \ge 3$; c) and d) are equivalent, only one is written as a less than statement and the other is the reversed greater than statement which is equivalent so both have the same solution set, because you are multiplying both sides by -1 to get $x \ge 3$.

Not all inequalities are linear. Consider the inequality $x^2 - 4x + 3 \le 0$.

- ***Find the boundary point given that it comes from {1, 2, 3, 4}.***

  Answer: There are two boundary points; both 1 and 3 make the inequality equal 0.

- ***Use the boundary point to find the solution to the inequality. Justify your answer.***

  Answer: The solution is $1 \le x \le 3$ because when values between 1 and 3 (for example, 2) are used for *x*, the inequality becomes a true statement. For values less than 1 or more than 3, the inequality will be false.

- ***How would you write a symbolic description for $x^2 - 4x + 3 > 0$?***

  Answer: $x < 1$ or $x > 3$.

Sample Assessment Items

After completing the lesson, students should be able to answer the following types of questions. If students understand the concepts involved in the lesson, they should be able to answer the following questions without using the TNS activity.

1. Consider the inequality $x > 7$. Determine whether each value of *x* shown in the table makes the inequality true. Select yes or no for each value.

	Yes	**No**
22		
–7		
13		
5		
–39		

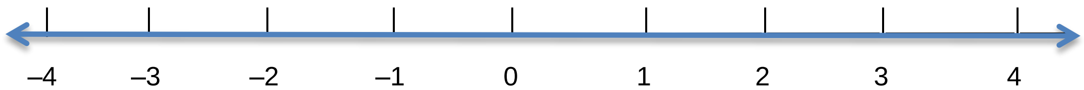
Smarter Balance Assessment Consortia Practice Test Scoring Guide, Grade 6 Mathematics, 5/1/15 #1806

Answer:

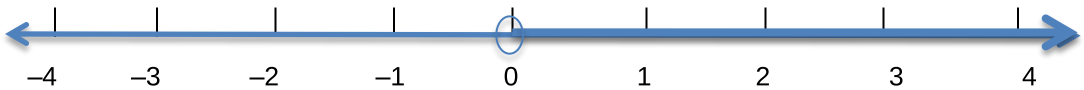
	Yes	**No**
22	√	
–7		√
13	√	
5		√
–39		√

2. Let *n* be an integer. Tracy claims that -*n* must be less than 0. To convince Tracy that his statement is only sometimes true:

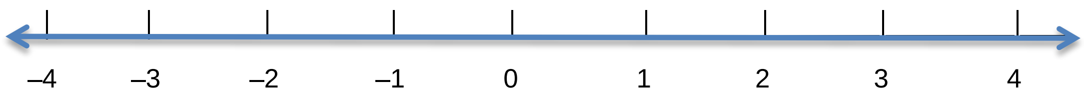
   a. Shade *n* on the number line so that the value of –*n* is less than 0.



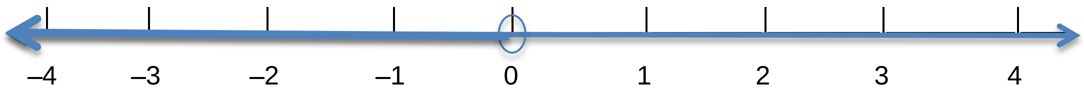
Answer:



   b. Shade *n* so that the value of –*n* is greater than 0.



Answer:

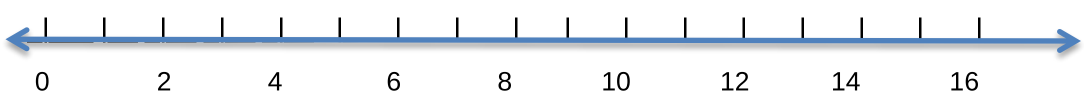


Adapted from Smarter Balance Assessment Consortia Practice Test Scoring Guide, Grade 6 Mathematics, 5/1/15 #1959

3. A boat takes 3 hours to reach an island 15 miles away. The boat travels:

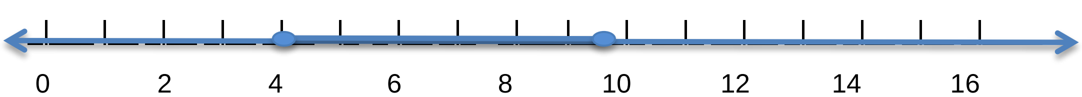
   - at least 1 mile but no more than 6 miles during the first hour
   - at least 2 miles during the second hour
   - exactly 5 miles during the third hour.

   Shade the range of miles the boat could have traveled during the second hour, given the conditions above.



Adapted from Smarter Balance Assessment Consortia Practice Test Scoring Guide, Grade 6 Mathematics, 5/1/151798 SMB

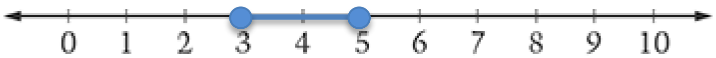
Answer: If in the first hour the boat travels the minimum 1 mile, then in the second hour it must travel 9 miles, but no more than 9 miles. If in the first hour the boat travels 2 miles, then in the second hour it must travel 8 hours. If the boat travels the maximum of 6 hours in the first hour, then it travels 4 miles in the second hour.



4. Graph the solution set for $3 \le x \le 5$ on the number line below.

NAEP, Grade 8. 2013

Answer:



5. What are all the whole numbers that make $8 - \_\_\_ > 3$ true?

   a. 0, 1, 2, 3, 4, 5

   b. 0, 1, 2, 3, 4

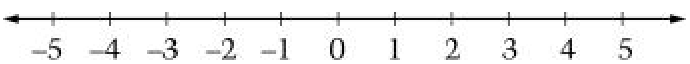
   c. 0, 1, 2

   d. 5

NAEP. Grade 8, 2003

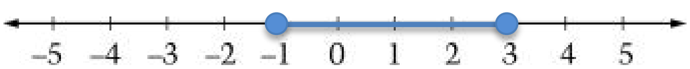
Answer: b

6. On the number line below, shade the part of the line that shows the set of all numbers greater than or equal to -1 and less than or equal to 3.



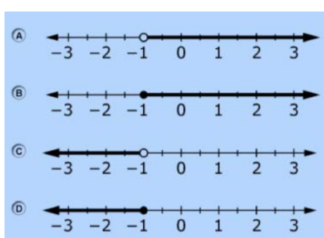
NAEP, Grade 8, 2003

Answer:



7. Which number line shows the solution to the inequality $-3x - 5 < -2$?

a.
b.
c.
d.



Smarter Balance Assessment Consortia Practice Test Scoring Guide, Grade 7 Mathematics, 5/1/15 1838

Answer: a

Student Activity Solutions

In these activities, you will use solution-preserving methods to solve inequalities. After completing the activities, discuss and/or present your findings to the rest of the class.

Activity 1 [Page 2.2]

1. What would you say to each of the following students?

   a. Milo: "You can subtract any number from both sides of an inequality, and the solution set for the new inequality will be the same as the solution set for the original inequality."

   Answers will vary. Milo is correct because adding a negative number is the same as subtracting, so the direction of the inequality will not change and the solution set will remain the same.

   b. Mai: "Multiplying both sides of an inequality by a negative number reversed the order of the inequality sign."

   Answers may vary. Mai is right, because for the inequality $x < 3$ *, multiplying both sides by* -2 *yields* $-2x > -6$ *. If* $x = 5$ *, you can think about what number added to* -6 *will equal* $-2(-5) = 10$ *-, and the number will be 16;* $-6 + 16 = (-2)(-5)$ *Since we added a positive number to -6 so that it was equal to* $-2(-5)$ *or 10, switching the order (direction) of the inequality sign makes a true statement.*

   c. Wu: "If you divide both sides of an inequality by the same number, the solution set for the new inequality will be the same as the solution set for the original inequality."

   Answers may vary: Wu is not correct because if you divide by a negative number, the direction of the inequality reverses. Also you cannot divide by zero.

   d. Petra: "Because the product of two negatives is a positive, when you multiply or divide a set of negative numbers by a negative number, the resulting numbers are 'flipped' to the other side of the number line."

   Answers will vary. Petra is correct although they will not land the same "distance" from 0 unless the multiplier is –1.

2. Use pages 1.7 and 2.2 of the TNS activity to help decide whether each of the following statements is true if $x < -7$. Describe what operation you could do on both sides of the inequality $x < -7$ to keep the solution set of the new inequality the same as the original solution set.

   a. $x + 3 < -4$ b. $3x < -21$ c. $x + 3 < -4x - 2 < -9$

   d. $-3x > 21$ e. $-3x < 21$ f. $-7 > x$

   Answer: a) adding 3 to both sides of the inequality will keep the solution set as $x < -7$ *; b) multiplying both sides by 3 will keep the solution set the same,* $x < -7$ *; c) taking 2 away from both sides will keep the solution set the same,* $x < -7$ *; d) multiplying both sides by* -3 *will reverse the direction of the inequality and keep the same solution set; e) not true, multiplying both sides of the inequality will change the solution set to* $x > -7$ *; f) this is the same inequality as* $x < -7$ *, just reversed.*

Activity 2 [Page 2.2]

1. Consider the inequality $7-2x \geq -5$. Find the solution set by

   a. finding the point of equality and testing a point on either side to determine the order of the inequality describing the solution.

   Answer: Thinking 7 minus what makes –5 means the missing addend is –12. What times –2 makes –12 means the missing factor is 6. Testing points:

$x=6$	Test value	$7-2x$	Is it >-5?
>6	8	$7-2(8)=7-16=-9$	False $-9<-5$
<6	0	$7-2(0)=7-0=7$	True $7>-5$

   The inequality was greater than or equal, so the solution is $x \leq 6$.

   b. using a solution- and/or order-preserving move on both sides of the inequality.

   Answer: Add a -7 to both sides to produce $-2x \geq -12$. Multiply both sides by $-\frac{1}{2}$ or divide by -2 to produce $x \leq 6$, remembering to change the order of the inequality.

2. Latashia suggested a new strategy.

   a. She added $2x$ to both sides of the inequality. What was her result? Would the solution or order change? Why or why not?

   Answer: $7 \geq -5+2x$; the order would not change nor would the solution, because you can add or subtract the same quantity to both sides of an inequality and still preserve the original solution set.

   b. What do you think her next step could be?

   Answers may vary. She could add 5 to both sides to get $12 \geq 2x$, then divide both sides by 2 to get $6 \geq x$.

   c. What is an advantage of Latashia's method?

   Answer: The multiplier of x is not negative, so you do not have to worry about changing the direction of the inequality, i.e., preserving order.

2. Why is it important to look for efficient and even elegant solutions?

   Answers will vary. One of the reasons is to save time to spend more time thinking about harder things. Another reason is that it reduces the chance of computational errors—i.e., not having to use the distributive property of multiplication over addition in a situation such as $21=5(x-3)+2(x-3)$.

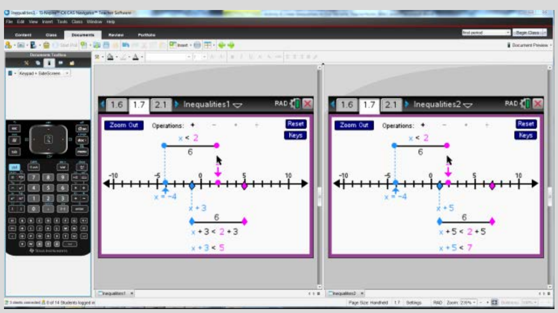
Part 2b, Pages 1.7 and 2.2

Focus: Develop a sense of "order- and solution-preserving" moves that can be applied to inequalities and how these moves relate to the solution set for an inequality.

Use Parts 2b and 3b with students who have not yet learned to add, subtract, multiply, or divide negative numbers.

Class Discussion

Teacher Tip: Working with a partner provides opportunities for students to compare solution sets for different inequalities. The teacher also can demonstrate two solution sets by creating a copy of the TNS activity with a new name, then using the multiple screen option on the software to show both screens at the same time. (Activate the multiple screen option by right clicking on the second activity at the bottom of the screen and selecting new vertical group.).



Have students…	**Look for/Listen for…**
Go to page 1.7. Work with a partner. Drag the blue dot (x) to 1 and the pink dot to 4.	
• ***What values are associated with the two diamonds on the number line?***	Answer: The blue diamond is at 6, and the pink diamond is at 9.
• ***What do the 3s related to the two segments represent?***	Answer: The 3s indicate the length of the segment or the distance between 1 and 4 and between 6 and 9.
• ***Remember if $a < b$ (here $x < 4$), then some positive number c will make $a + c = b$. Using the graph and $x = 1$, is there a positive number you can add to $x = 1$ to get to 4? to $x + 6$ to get to 9? In both cases, write the arithmetic sentence that supports your answer. How does this indicate whether the relationship is > or <?***	Answer: in both cases you would add 3: $1 + 3 = 4$ and $(1+5)+3 = 9$. If $a < b$, then you can add a positive number to a to equal b, which happens in both of these cases.

Class Discussion (continued)

• ***How are the two inequalities on the screen related?***	Answer: If you add 5 to both sides of $x<4$, you will get $x+5=9$.
Go to page 2.2. Create $x<3$ by selecting the space before 3 in the inequality and then select the right side and enter 3. Your partner should create $x+5<8$ by selecting the space before 3 in the inequality, then the space after the **x** ***and enter 5, then the space on the right side of the inequality and enter 8. Show*** **graph*.***	
• ***How do the graphs of the two inequalities compare? What does this tell you about the solution sets for the two inequalities?***	Answer. The graphs are the same, and so the solution sets will be the same, $x<3$.
• ***Create $x<10$ and $x-4<6$. Compare the solution sets for the two inequalities.***	Answer: They are the same, $x<10$.
• ***Make a conjecture about how the solution sets will change when you add or subtract the same number on both sides of an inequality. With your partner, create two examples to support your conjecture.***	Answers may vary. A conjecture might be that solution sets are preserved, including the order of the inequality.
• ***Create several inequalities that are stated as greater than and compare the solution sets. Make a conjecture about how the solution set for inequalities that are greater than will change when you add or subtract the same value on both sides of the inequality.***	Answer: The solution sets do not change.
Return to page 2.2. Select the multiplication option. With your partner, create the inequalities $x<3$ and $2x<6$ and **Show Graph**.	
• ***Is the solution set what you would expect? Why or why not?***	Answers may vary. Typically, students would think both inequalities have the same solution set.

Class Discussion (continued)

- ***For each inequality, give an example of one number that is in the solution set and one number that is not in the solution set. Show why your examples are correct.***

Answers will vary. 0 is in the solution set for both; $0<3$ and $2(0)=0<6$ because 0 is the left of both 3 and 6 on the number line. 8 is not in the solution set for either because 8 is to the right of both 3 and 6 on the number line.

Test	$x<3$	$2x<6$
0	In: $0<3$	In: $2(0)=0<6$
6	Not in: $6>3$	Not in: $2(6)=12>6$

What would you say to each of the following?

- ***Milo: You can subtract any number from both sides of an inequality, and the solution set for the new inequality will be the same as the solution set for the original inequality.***

Answers will vary. Milo is correct because adding a negative number is the same as subtracting, so the direction of the inequality will not change and the solution set will remain the same.

- ***Mai: Multiplying both sides of an inequality by a positive number kept the order of the inequality sign.***

Answers may vary. Mai is right because you can always find a positive number to add to the smaller side to equal the larger so the order stays the same.

Use pages 1.7 and 2.2 of the TNS activity to help decide whether each of the following statements is true if $x<7$. Describe what operation you could do on both sides of the inequality $x<7$ to keep the solution set of the new inequality the same as the original solution set.

a. $x+3<10$ ***b.*** $3x<21$

c. $x-2<5$ ***d.*** $\frac{1}{2}x<14$

e. $\frac{2}{3}x<\frac{14}{3}$ ***f.*** $7>x$

Answer: a) adding 3 to both sides of the inequality will keep the solution set as $x<7$; b) multiplying both sides by 3 will keep the solution set the same, $x<7$; c) taking 2 away from both sides will keep the solution set the same, $x<7$; d) multiplying both sides by 2 will not keep the same solution set but rather $x<28$; e) multiplying both sides of the inequality by $\frac{3}{2}$ will give $x<7$; f) this is the same inequality as $x<7$, just reversed.

Part 3b, Page 2.2

Focus: Find the solution for a linear inequality in one variable using solution-and operation-preserving moves for inequalities.

Class Discussion

Have students…	Look for/Listen for…
Select Show Graph.	
• ***Drag the dot. Does the shading stop at 0? at 1? Why or why not?***	Answer: No, because a fraction like $\frac{1}{3}$ is larger than 0, but $3\left(\frac{1}{3}\right)=1<2$.
• ***At what value of*** **x** ***will the expression on the left be equal to 2? Explain how you found your answer.***	Answer: Thinking, what number times 3 will equal 2, gives the point of equality as $\frac{2}{3}$.
• ***Describe the solution set for the inequality $3x<2$. How do you know you have all the possible solutions?***	Answer: The solution set is the set of all of the numbers less than $\frac{2}{3}$. You have them all because using any number greater than $\frac{2}{3}$ for x will produce a false statement.
• ***Select*** **Show Graph** ***then*** **Zoom Out.** ***How does the graph support your answer to the first question?***	Answer: The graph shows all of the numbers less than $\frac{2}{3}$ are the solution, and those numbers keep going to the left of $\frac{2}{3}$.

Previously, students were asked how to change an inequality with just the variable on one side into a new inequality with a number added to or multiplied by the variable. The following question asks students to start from a more complicated inequality and find the solution.

Find the point of equality, the boundary point, for each of the expressions by using what you know about the highlight method for solving equations. Then find the direction of the inequality by testing points. Use the TNS activity to check your solution.

• $x-4=5$; $x-4>5$

Answer: The answer to the question "what minus 4 equals 5?" implies that $x=9$. Checking a point from each side of the boundary:

The inequality is greater than, so the solution set

$x=9$	Test value	$x-4$	Is it <5?
>9	10	$10-4=6$	True $6>5$
<9	8	$8-4=4$	False $4<5$

is $x>9$.

Class Discussion (continued)

- ***$2x-4=5$; $2x-4\geq 5$***

Answer: What minus 4 equals 5 implies that $2x=9$, so $x=\frac{9}{2}$. Testing a point on either side:

$x=\frac{9}{2}$	Test value	$2x-4$	Is it >5?
$>\frac{9}{2}$	5	$2(5)-4=6$	True 6 > 5
$<\frac{9}{2}$	2	$2(2)-4=0$	False 0 < 5

The inequality is ≥ 5 so the solution set is $x\geq\frac{9}{2}$.

- ***$5=2x+4$; $5<2x+4$***

Answer: What plus 4 equals 5 implies that $2x=1$, so $x=\frac{1}{2}$. Testing a point on either side:

$x=\frac{1}{2}$	Test value	$2x+4$	Is it >5?
$<\frac{1}{2}$	0	$2(0)+4=4$	False 4 < 5
$>\frac{1}{2}$	3	$2(3)+4=10$	True 10 > 5

The inequality is greater than 5 so the solution set for $5<2x+4$ is $x>\frac{1}{2}$.

Create each of the inequalities. Use the TNS activity to find the solution, then identify a value and show why it is in the solution set. Describe the order-preserving moves you could use to find the solution.

- ***$x-4>3$***

Answer: $x>7$ by adding 4 to both sides. Examples will vary: 8 because $8-4=4>3$.

- ***$6>x-3$***

Answer: $x<9$ or $9>x$ by adding 3 to both sides. Examples will vary: 3 because $6>3-3=0$.

- ***$4x\geq 12$***

Answer: $x\geq 3$ by multiplying both sides by $\frac{1}{4}$ or dividing by 4. Examples will vary: 4 because $4(4)=16\geq 12$ 4(4) = 16 ≥ 12.

Class Discussion (continued)

- ***$1 > x - 4$***

Answer $x < 5$ or $5 > x$ by adding 4 to both sides. Examples will vary: 4 because $1 > 4 - 4 = 0$.

Consider the inequality $3x - 2 > 7$. Find the solution set by

- ***finding the point of equality and testing a point on either side to determine the order of the inequality describing the solution.***

Answer: Thinking what number minus 2 will be 7 gives $3x = 9$. Thinking what number multiplied by 3 gives 9 leads to $x = 3$. Testing the point:

$x = 3$	Test Value	$3x - 2$	Is it > 7?
< 3	2	$3(2) - 2 = 4$	False $4 < 7$
> 3	4	$3(4) - 2 = 10$	True $10 > 7$

The inequality was greater than so the solution set is $x > 3$.

- ***using solution-preserving moves on both sides of the inequality.***

Answer: Adding 2 to both sides gives $3x > 9$; dividing both sides by 3 (a positive number) gives $x > 3$ for the solution.