Solving Systems by Graphing

TEACHER NOTES

ENHANCED FOR THE ESSENTIAL MATH TEACHING PRACTICES

These Teacher Notes were enhanced to demonstrate how to integrate the Eight Essential Mathematics Teaching Practices into a lesson, with specific examples for when and how to address students' questions and work in class. Look for the Math Teaching Practices, which have been noted in italics, throughout the lesson to encourage student thinking and create effective mathematical discourse.

Math Objectives and Learning Goals

Establish mathematics learning goals to focus learning. *Prior to doing this lesson with students, you should establish the math objectives – which state what students will do as a part of the lesson – and the learning goals – that state what the students will understand as part of the lesson. Knowing those goals and objectives will help you focus your questions and guide discussions with students.*

- ***Learning Goal:*** *Students will understand that the solution to a system of linear equations occurs at the point of intersection of the graphs of the two linear equations and is indicated by the ordered pair the represents the point.*
- **Math Objective:** Students will visualize a solution to a system of equations.
- ***Learning Goal:*** *Through this exploration, students will find that solutions to linear systems of equations are defined at exactly one point, no points or an infinite number of points; and that the slope of the lines can help determine if a solution to the system exists. Students can also think about the solution to a system of linear equations in terms of the solution of a one variable, that is, linear equation. The solution to $x+3=0$ is the intersection of the graph of the equation $y=x+3$ with the line $y=0$. This can be thought of as finding the solution to a system of the two linear equations $y=x+3$ and $y=0$.*
- **Math Objective:** Students will explore a system that intersects in one point and identify the ordered pair that represents the solution.
- ***Learning Goal:*** *Students should recognize situations in which the use of technology is appropriate in supporting their learning of a concept. They should understand the capabilities and limitations of their tools, so that they can reason appropriately when using them.*
- **Math Objective:** Students will use appropriate tools strategically.

Principles to Actions:

This lesson includes a guide to using the Essential Mathematics Teaching Practices, as described in *Principles to Actions: Mathematics Success for All.*

Tech Tips:

- This activity includes screen captures from the TI-Nspire™ CX handheld. It is also appropriate for use with the TI-Nspire™ family of products including TI-Nspire™ software and TI-Nspire™ App for iPad®. Slight variations to these directions may be required if using other technologies besides the handheld.
- Watch for additional Tech Tips throughout the activity for the specific technology you are using.
- Access free tutorials at http://education.ti.com/calculators/pd/US/Online-Learning/Tutorials

Vocabulary

- solution
- system of equations
- linear equations

Lesson Files:
Student Activity
- Solving_Systems_by_Graphing.pdf
- Solving_Systems_by_Graphing.doc

TI-Nspire document
- Solving_Systems_by_Graphing.tns

About the Lesson

In this activity, students will explore the solution to a system of linear equations. They will move a point on the x-axis to change the x-coordinate of a point on each of the lines. They will determine when the point on each line is a solution to the system of equations. This method could provide visual cues to help students determine if their algebraic solution is reasonable.

Implement tasks that promote reasoning and problem solving: *This lesson should be included as an introduction to solutions to systems of linear equations. It is recommended that students explore the concepts introduced in this activity prior to instruction such as definition of systems of equations. Allowing students the time to visualize, think about and discuss the concepts for systems of equations allows them to develop deeper meaning of the mathematics.*

Students may work independently or in small groups for this lesson. It is important for the teacher to monitor students' work, to ask questions throughout the lesson and to include class discussion in order to ensure student understanding.

TI-Nspire™ Navigator™ System

- Use Class Capture to observe students' work as they proceed through the activity.
- Use Live Presenter to have a student illustrate how he/she moved the point on the *x*-axis.

Activity Materials

Compatible TI Technologies: TI-Nspire™ CX Handhelds, TI-Nspire™ Apps for iPad®, TI-Nspire™ Software

What's Next?

Follow-up to this lesson may include the following activities from Math Nspired:

- What is a Solution to a System?
- How Many Solutions to the System?
- Solving Systems by the Elimination Method

Discussion Points and Possible Answers

Tech Tip: If students have difficulty dragging a point, check to make sure that they have moved the arrow until it becomes a hand getting ready to grab the point. Also, be sure that the word *point* appears. Then press ctrl to grab the point and close the hand. When finished moving the point, press esc to release the point. Once a function has been graphed, the entry line can be shown by pressing ctrl G or tab.

Teacher Tip: Instructions for creating the TI-Nspire document, *Solving_Systems_by_Graphing.tns,* are available at mathnspired.com.

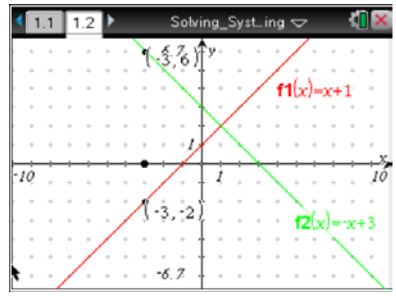
Elicit and use evidence of student thinking: *Do a quick assessment of students' understanding of solutions to linear equations. You may ask students to explain what it means to be a solution to an equation.*

- *On the board, represent the equation x+3=8 on a number line to help students visualize the solution in one dimension.*
- *Next, graph the line y= x+3 and ask students to explain the following:*
    - *The meaning for y=0. Students should be able to explain that the x-intercept is the point of intersection of the graph of the equation with the line y=0. They should also be able to explain that the x-intercept is the point at which the y-value for the equation is equal to 0.*
    - *Solve for y=8. Students should be able to use substitution to solve algebraically or use a table or graph to show the solution. When sharing student solutions, one that needs to be stressed is that the solution to the equation y=x+3 when y=8 is the same as finding the point of intersection of the graph of the equation with the line y=8.*

Students may be able explain the solution of a linear equation graphically or algebraically. Be sure that students have an opportunity to understand solutions from both perspectives for this lesson. It may be helpful for students to explain by showing others using a graph in Scratchpad. Press to access

Scratchpad.

Move to page 1.2.



Questions 1–3 refer to the system of equations graphed in the TI-Nspire document: **f1**(x) = x + 1
f2(x) = -x + 3

Move the point that is on the *x*-axis to the left or right as needed.

1. Move the point so that *x* = -2 in both sets of coordinates. Is either of these ordered pairs a solution to the given system of equations? Justify your answer.

   Answer: When *x* = -2, the value of the function **f**1 is -1. When *x* = -2, the value of the function **f**2 is 5. Since the function values are not equal, this is not a solution to the system of equations.

2. Move the point so that *x* = 3 in both sets of coordinates. Is either of these ordered pairs a solution to the given system of equations? Justify your answer.

   Answer: When *x* = 3, the value of the function **f**1 is 4. When *x* = 3 the value of the function **f**2 is 0. Since the function values are not equal, this is not a solution to the system of equations

Pose purposeful questions: *Be sure to ask questions as they relate to student work. Outlined here are some scenarios that are likely to happen as you start this lesson. Be mindful of the questions you ask so that they do not lead directly to an answer. Allow students time to think and reason. Their responses will help identify the route you should take with the next question.*

Possible Student Response	***Possible Teacher Question***	***Question Type***
Students think that the points located at x=-2 or x=3 are solutions to the system of equations. They probably realize that these x-values represent points on the line, but they may not recognize the difference between this and the	*On the graphs of f1(x) and f2(x), what happens at x=0 and at y=0?*	*Gathering information*

solution.		
Students think that the points located at x=-2 or x=3 are solutions to the system of equations. They probably realize that these x-values represent points on the line, but they may not recognize the difference between this and the solution.	*How are the coordinates on the lines related at x=-2 and at x=3?*	*Gathering information*
Students may struggle to identify differences between the coordinates of the points at x=-2 and x=3 on the two graphs.	*Have them make note of the coordinates on f1(x) and f2(x) at x=-2 and at x=3.* *What is the same and what is different about those points?*	*Probing thinking*
Students may think that a solution to each linear equation represents the solution to the system.	*How would you show the solution to a linear equation on a graph?*	*Making the mathematics visible*
Reflecting back on the start of the lesson, students can find the solution to y=x+3 and y=8. But they struggle with finding the solution to a more complicated system of equations.	*What do you know about solutions to linear equations that could help you find the solution to this system of linear equations?* *How can you adapt your method for finding the solution to y=x+3 and y=8 to a new situation (maybe using y=x and y=4-x)?*	*Encouraging reflection and justification*
Reflecting back on the start of the lesson, students can find the solution to y=x+3 and y=8. But they struggle with finding the solution to a more complicated	*The solution to a linear equation (such as x+3=0) is the intersection of y=x+3 with the line y=0. For two lines in a system of equations, how do you think you might describe the*	*Making the mathematics visible*

system of equations.	*solution?*	

Facilitate meaningful mathematical discourse: *In asking these questions, you should be thinking about how you want to sequence students' answers for explanation to the whole class. The order in which these questions are posed is the recommended order in which students should explain to the class. Be sure that students have time to internalize each response before moving on to the next point.*

Question to Students	***Teacher Actions***
How can you show the solution to a linear equation on a graph?	*Listen for students to respond that the solution to a linear equation is the single point at which the graph intersects the x-axis, or y=0.*
What do you expect the solution of any system might be; knowing that the solution to each linear equation has already been defined above?	*Listen for students to respond that the solution may be the point where the graphs intersect each other. Ask students to clarify their meaning and explain how the graphical solution for the solution to y=x+3 and y=8 corresponds to the analytical solution. Groundwork is being set to consider parallel and coincident lines. Be prepared for students to consider those options.*
Graphically, if the solution to the system is the intersection of the two lines, what can you conclude about that point of intersection in terms of each equation?	*Listen for students to say that both lines share the same coordinate; that equations are equal at this point.*

3. What is the solution to the system pictured? Explain how you know.

   Answer: The solution to the system pictured is (1, 2). These are the coordinates of the point that is the intersection of the two lines. That is, when $x = 1$, the y-coordinate on each line is 2.

Tech Tip: Students can change the equation of the lines graphed by moving the cursor on top of the label for the equation and clicking twice to make edits to the equation. Students could also add the new functions introduced, **f**3(x) and **f**4(x), in the function entry line.

Tech Tip: To add new functions for **f**3(x) and **f**4(x), students can show the function entry line by double-tapping the page.

4. Jean told the class that she knew of another system that had the same solution as the system pictured in the graph. Her system is

   f1(x) = x + 1

   f3(x) = 4x – 2

   Bryan argued that he thought that the system with the same solution as the system pictured in the graph was the following:

   f2(x) = -x + 3

   f4(x) = 2x

   Who is right? Explain your reasoning.

   Answer: Both students are correct. The solution to both systems is the ordered pair (1, 2). A graph would show that **f**1(x) and **f**3(x) intersect at the point (1, 2) and **f**2(x) and **f**4(x) also intersect at (1, 2).

5. Find the solution to the following system graphically. Show the necessary work to check the solution.

   f1(x) = x + 1

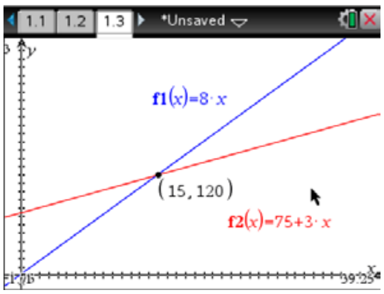
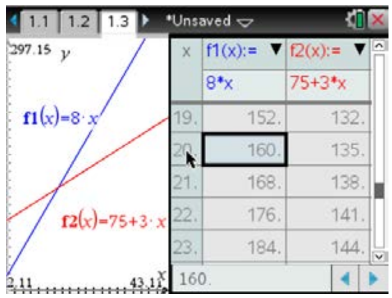
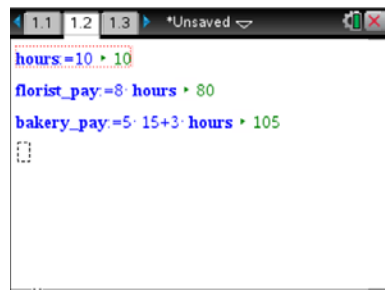
   f2(x) = -x – 3

   Answer: The solution to the system is (-2, -1). These are the coordinates of the point that is the intersection of the two lines. That is, when x = -2, the y-coordinate of each line is -1.

Use and connect mathematical representations. *This is an opportunity for students to make a connection to the mathematics through a problem in context. For example:*

Jerri works at a flower shop making $8 per hour. She has been offered a job at bakery in which she will make $15 per day, plus another $3 per hour. If Jerri can only work at total of 20 hours Monday through Friday each week, which job would allow her to earn more money? If she could only work 10 hours, which job would be better? What if she could work 30 hours?

Students could solve this problem using any one of the methods shown.

Graphical	Tabular	Analytic
		

Facilitate meaningful mathematical discourse. *This question is important for emphasizing students' ability to critique the reasoning of others. Allow your students to discuss this question in small groups to give them an opportunity to analyze and compare the ideas of others.*

Practices for effective discourse	***Teacher response/ actions***
Anticipating student responses	*Prior to the lesson, think about the types of ways that your students may try to solve this problem and how you may respond. It is easier to approach the lesson with a plan for the discussion than attempting to do it on the spot.*
Monitoring student work	*As students work through the previous problem, make some observations about student work. Rather than providing a direct path to an answer, allow students time to reason on*

	their own or to discuss possibilities with another student. If a student is stuck, you may ask questions to help them get started.
Selecting students to present work	*As you observe students' work, identify solutions that can be presented to the class or group. Ask students to present in the following order.*
Sequencing students' responses	*Select student solutions to help build ideas:* *1. Solution shown on graph* *2. Solution shown in table* *3. Solution explained verbally* *4. Solution found using algebraic methods in order of a) substitution; b) elimination*
Connecting students' responses to key ideas	*Ask students to discuss the following:* - *What can you conclude about the relationship between the equation of the lines and the solution of the system?* - *In terms of this problem, explain the meaning of the intersection point on the graphs.*

Pose purposeful questions. *Students may get stuck on finding the solution analytically. You may want to ask:*

Question	***Question Type***
How do you find the solution to a linear equation? *How can you use that knowledge and what we have discussed in class today to help you find the solution to this system of equations?*	*Making the mathematics visible*
How would you convince another student that this is the solution to the system?	*Encouraging reflection and justification*

Support productive struggle in learning mathematics. *It is important to ask questions and to help make mathematical questions. However, it is just as important to allow students some time to grapple with the mathematics. Allowing time for students to work alone in small groups will allow them time to* *internalize concepts being discussed in class. Be mindful of the time you give students to respond, of the time between the subsequent questions you ask and of the amount of "help" you provide students as they form ideas about the concepts.*

TI-Nspire™ Navigator™ Tech Tip*: Have students graph a system of equations for which the solution is (2,3). Use Class Capture to compare student work. Allow students to compare their equations and to discuss why the graphs or equations are different, even though the solutions are the same.*

Elicit and use evidence of student thinking. *Before students are ready to move on in this lesson, be sure students understand how the solution to the system of equations is found graphically and analytically.*

- *Have students change the slope and y-intercept of each of the graphs to explore how the solution changes.*
- *Ask students to graph a system of equations for which the solution is (2,3). Allow students to compare their equations and to discuss why their equations are different, even though the solutions are the same.*
- *Ask students to explain to a partner how they know how to find the solution to a system of equations using a graph and without using a graph.*
- *If students show difficulty with the concept of a solution, be prepared with additional questions that may clarify ideas*

6. Find the solution to the following system graphically. Show the necessary work to check the solution.

   f1(x) = -x + 3

   f2(x) = -x – 3

   Answer: This system has no solution. The lines do not intersect. The equations have the same slope and a different y-intercept. Consequently, the graphs of these lines are parallel. They do not have any points in common.

Teacher Tip: *This problem provides an opportunity to have students discuss how to use their technology effectively.*

- *Is it possible that the solution is not in the viewing window?*
- *Could it be possible for a system of linear equations to appear graphically that they have no solution, but they really do?*

Use and connect mathematical representations: *Have students work in small groups to look for the solution to this system of equations.*

- *Use tables, graphs or algebraic analysis to show whether or not there is solution to this system of linear equations.*
- *What do you notice about the equations of the lines for this system? How can you use evidence from a table, graph or equation to tell whether or not this system has a solution?*

7. How does the solution to the system in problem 6 compare to the solution of the system in the previous problems? Justify your answer.

   Answer: This system does not have a solution, whereas the other systems had one solution, a point of intersection.

Facilitate meaningful mathematical discourse. *At this point in the lesson, students should be forming key ideas through the exploration and discussion. Providing an opportunity to allow students to participate in meaningful discourse is important to helping solidify the key ideas in the lesson. Ask students to discuss the following in their smaller groups. As the students discuss in their small groups, make note of which groups should share their thinking with the larger group. Student work should be shared in the order of the questions listed here.*

- *What can you conclude about the relationship between the equation of the lines and the solution of the system?*
- *Will there always be a solution to a system of equations? How do you know?*
- *Are there lines that appear to have no solution by looking at the graphs, but they actually do? What can you do to find the solution?*
- *Will there always be a solution to a system of equations? How do you know?*
- *Can another system of equations have the same solution as the equations presented in Question 1?*
- *How can you prove that there is or is not a solution to a system of equations?*

Build procedural fluency from conceptual understanding: *Allow time for students to explore different systems of equations in order to gather evidence to support their thinking about the following questions.*

- *Without using a graph or table, what do you know about the lines y=2x-1 and y=2x?*
- *Without using a graph or table, how would you solve this system of equations? Explain your reasoning to another student. y= -2x and y= x+4.*
- *How would a graph support your solution?*

Wrap Up

Upon completion of the discussion, ensure that students understand:

- What is meant by the reference to an ordered pair as a solution to a system of linear equations?
- The point of intersection of two lines is the solution to that system of linear equations.

Assessment

Solve the system that follows graphically. Explain why the ordered pair you give as the answer is the solution to the system.

$y = 2x – 1$
$y = –x + 5$

Answer: When $x = 2$, y is equal to 3 in each equation. (2, 3) is the point of intersection of the graphs of the two equations.