These Teacher Notes were enhanced to demonstrate how to integrate the Eight Essential Mathematics Teaching Practices into a lesson, with specific examples for when and how to address students' questions and work in class. Look for the Math Teaching Practices, which have been noted in italics, throughout the lesson to encourage student thinking and create effective mathematical discourse.

Math Objectives and Learning Goals

Establish mathematics learning goals to focus learning. *Prior to doing this lesson with students, you should establish the math objectives – which state what students will do as a part of the lesson – and the learning goals – that state what the students will understand as part of the lesson. Knowing those goals and objectives will help you focus your questions and guide discussions with students.*

- ***Learning Goal:*** *Students will understand how to create, interpret and analyze a data set by use of the five number summary and the associated boxplot.*
- ***Learning Goal:*** *Students will understand and explore the interrelationship between a dot plot and the boxplot for a data set.*
- **Math Objective:** Students will describe the center, spread and the shape of a data set using the dot plot.
- **Math Objective:** Students will describe the overall distribution of a data set using a five number summary - center, spread and the shape of its boxplot.
- **Math Objective:** Students will recognize and identify properties of distributions such as symmetry, skewness, and bimodality from dot plots and boxplots.
- ***Learning Goal:*** *Students will understand what an outlier is and how changing values of specific data points may affect the center, spread and/or shape of a distribution.*
- **Math Objective:** Students will be able to identify outliers, and will recognize that moving data points may affect that quartile values.
- **Math Objective:** Students will reason abstractly and quantitatively about trends and patterns in data distributions using box plot and dot plot representations.
- ***Learning Goal:*** *Students should recognize situations in which the*

Principles to Actions:

This lesson includes a guide to using the Essential Mathematics Teaching Practices, as described in *Principles to Actions: Mathematics Success for All.*

Tech Tips:

- This activity includes class captures taken from the TI-Nspire™ CX handheld. It is also appropriate for use with the TI-Nspire™ family of products including TI-Nspire™ software and TI-Nspire™ App. Slight variations to these directions may be required if using other technologies besides the handheld.
- Watch for additional Tech Tips throughout the activity for the specific technology you are using.
- Access free tutorials at http://education.ti.com/calculators/pd/US/Online-Learning/Tutorials

use of technology is appropriate in supporting their learning of a concept. They should understand the capabilities and limitations of their tools, so that they can reason appropriately when using them.

- **Math Objective:** Students will use appropriate tools strategically. (CCSS Mathematical Practice)

Lesson Files:

Student Activity

- Boxplots_Introduction_Student.pdf
- Boxplots_Introduction_Student.doc

TI-Nspire™ document:
Boxplots_Introduction.tsn

Vocabulary

- boxplot
- skewness
- symmetry
- quartile
- skewed left
- outlier
- interval
- median
- dot plot
- interquartile range, IQR
- range
- distribution
- skewed right

About the Lesson

This lesson involves representing distributions of data using boxplots. As a result, students will:

- Develop the reasoning skills to recognize properties of a distribution from its boxplot and/or dot plot.
- Understand the relationship between individual data values and the five-number summary.
- Move data within a data set and observe the changes within the corresponding boxplot and/or dot plot to identify particular characteristics that most directly affect the five-number summary.

Implement tasks that promote reasoning and problem solving: *This lesson introduces students to the concept of representing data using boxplots. The activity allows students to reason abstractly and quantitatively about the relationship between the quartiles and the spread of data within the quartiles. Students are given a set of data (student heights) from which they must create different distributions using a set of constraints. Students analyze the distributions by looking at both the box plot and the dot plot of each so that they may reason about the shape, center, and spread of the data and how any relationships exist among the different distributions.*

At the start of the lesson, you may want to formatively assess your students' level of understanding of dot plots. Students should have a basic understanding of dot plots and know how to construct them from a set of data.

Allowing students the time to visualize, think about and discuss the concepts in this activity helps them to develop a deeper meaning of the mathematics. Students may work independently or in small groups *for this lesson. It is important for the teacher to monitor students' work, to ask questions throughout the lesson and to include class discussion in order to ensure student understanding.*

TI-Nspire™ Navigator™ System

- Send out the *Boxplots_Introduction.tns* file.
- Monitor student progress using Class Capture.
- Use Live Presenter to spotlight student answers.

Activity Materials

Compatible TI Technologies: TI-Nspire™ CX Handhelds, TI-Nspire™ Apps for iPad®, TI-Nspire™ Software

What's Next?

Follow-up to this lesson may include Math Nspired: Multiple Boxplots*, which allows students to look at parallel boxplots to compare data sets.*

Discussion Points and Possible Answers

Tech Tip: If students experience difficulty dragging the point, check to make sure that they have moved the arrow until it becomes a hand (☝) getting ready to grab the point. Also, be sure that the word *point* appears. Then press ctrl ☝ to grab the point and close the hand (✊).

Tech Tip: If students experience difficulty grabbing and dragging a point, have them tap and hold the desired point for a few seconds and then drag the point to the desired location.

Support productive struggle in learning mathematics: *Students may have difficulty meeting specific constraints that are given as prompts.*

Students may have trouble understanding that there are the same number of data points in each quartile or that the shape of the boxplot indicates the shape of the dot plot. For example, students may think that *a long whisker on the box plot means that the data is evenly distributed.*

Ask clarifying questions that help students to think about the constraints rather than giving cues or suggestions as to how to meet the constraints. Allow students time to explore with the data and to discover what happens on their own.

This lesson allows students an opportunity to explore data and distributions by observing how changes to one representation affect the other. Students may move points to satisfy a constraint only to discover that without careful thinking, a new move may undue the effect of the one before. Many students only discover through trial and error that to change the location of a median or a quartile; at least one point has to be moved past the median or quartile.

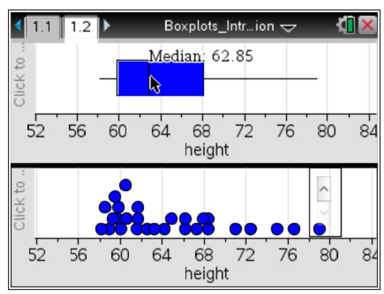
Elicit and use evidence of student thinking: *Your students may be familiar with collecting data that has to do with counting or measurements and representing it using dot plots. You will need to do a brief assessment before beginning this lesson to determine the facility the students have with dot plots. The data used in this activity is pre-loaded in the activity file so that you are able to focus the lesson on interpreting the data, rather than collecting and entering it. Prior to starting this lesson, it may be helpful to provide context for this lesson and to assess students' understanding of the characteristics of center and spread. As a group, measure the height of 10 students. Before the implementing the lesson, use the preloaded measurements to discuss the following with your students:*

- *How a dot plot represents the data*
- *How to find the median value for this sample – have students describe their methodology*
- *How students know whether or not the median is in the data set*
- *Why it is important for us to use the same units when measuring our data.*

Use and connect mathematical representations: *Before students start exploring the activity, help students understand how data is represented using a boxplot by drawing a boxplot from the data found at the start of this lesson. Students can model a living boxplot by having ten students hold cards with different values on the cards and lining up to model the data. Additionally, students should then use a number line diagram, on which students mark off the minimum, maximum and median values. For either representation discuss with students how to find the quartile values (as the median of each half of the data set) and mark these on the living boxplot or diagram. Once the extreme values, median and quartiles are recorded on the number line, construct the box plot. This exercise is important for students to understand prior to exploring box plots further. As you lead students through this example, be sure to encourage discussion and ask questions to help solidify the concepts for students.*

Strategy	***Example***
Encourage purposeful selection of representations.	*What information is lost when using a boxplot?* *What benefits are there when using a boxplot?* *When would you want to use either representation?*
Engage in dialogue about explicit connections among representations.	*What connections or similarities can you see between the dot plot and the box plot?* *What differences do you see between the dot plot and box plot?* *What information does a dot plot tell you that a box plot does not?* *What information does a box plot tell you that a dot plot does not?*
Alternate the direction of the connections made among representations.	*If you were given a dot plot, could you determine the box plot?* *If you were given the box plot, could you determine the dot plot?*

Move to page 1.2.



1. In the top work area, move the cursor over the boxplot from left to right, and identify the five summary percentiles of the boxplot. These five important statistical values are called the five-number summary.

<u>Answer:</u>

Min = 58.12 inches

Q1 = 59.9 inches

Median = 62.85 inches

Q3 = 68.1 inches

Max = 79 inches

Tech Tip: This document has an up arrow in the lower right work area of page 1.2 that can be used to return the data to its original state. If you change the data by mistake, select ctrl Z to return the data to the original state or to undo an action; select ctrl Y to redo an action.

Tech Tip: If you change the data by mistake, select the undo/redo icon to return the data to the original state or to undo an action; select and hold on the icon and select **Redo** to redo an action. Or use the up arrow in the dot plot area of page 1.2.

Tech Tip: To display each of the five-number summary values on the boxplot, the students should tap the line or region on the boxplot.

Teacher Tip: Caution students not to grab and drag the data points until instructed to do so or the answers given below will be incorrect. Be sure students understand how to move points. When a point has been selected, it will be darkened. To deselect a point, move the cursor to an open region and select or tap. To move a single point, select a point in the dot plot and move the darkened point by grabbing and moving the point from either plot. To move several points at once, select each one, grab and move just one selected point, and all of the selected points will move as a unit. To move the set of points in a quartile of the boxplot, select that interval in the boxplot, and the points will drag as a group.

2. a. Determine the longest interval in the box. How long is it?

Answer: The longest interval is between Q3 and the maximum.

79 – 68.1 = 10.9

b. Which interval do you think has the most data points? Explain your reasoning, and then check your answer by selecting each interval.

Sample Answers: They all contain the same number because each interval should contain 25% of the data. If there are 28 points, you would expect to see seven in each interval. In order to check your answer, you have to go back and forth between the boxplot and the dot plot because the points are so close together. For example, from the median to Q3 in the boxplot, it appears there are only six points, but the dot plot shows seven; in the upper quartile on the boxplot, there seem to be only six as well, but the dot plot shows all seven of the points.

Teacher Tip: Exceptions can occur in other distributions. Examples include 1) the number of data is not divisible by four or 2) duplicate data values near a quartile.

3. Consider the dot plot in the lower work area on the same page.
   a. Describe the center, shape, and spread of the distribution of heights.

   Answer: This is a right-skewed distribution with a center of approximately 63, with the data more spread out as the heights increase. Typical values fall between 60 and 68.

Teacher Tip: Median is an acceptable measure of center. Interquartile range or range is an acceptable student measure of spread.

   b. How is the shape of the distribution visible in the boxplot?

   Sample Answers: The skewness in the distribution is visible because the interval lengths generally increase from left to right.

Pose purposeful questions: *Students may be surprised that the size of the interval does not relate to the number of data points in the interval. Be sure that they understand that the interval between Q1 and the median has just as many data points as the interval between the median and Q3. Be mindful of the questions you ask so that they do not lead directly to an answer. Allow students time to think and reason. Their responses will help identify the route you should take with the next question.*

Teacher Question	***Question Type***
Describe the center, shape, and spread of the distribution of heights using the dot plot and the box plot.	*Gathering information*
Which interval do you think has the most data points? Why? *Explain what the Q1 and Q3 values tell about the data.*	*Probing student thinking*
Compare the similarities and differences between the box plot and the dot plot.	*Making the mathematics visible*

What information can you get from the dot plot that you cannot get from the box plot? *What would it mean if the box plot had no "whiskers"?*	*Encouraging reflection and justification*

4. a. Find the person with a height of 62.5 inches. Use the boxplot to describe how this person's height relates to the distribution as a whole.

   Answer: The person who is 62.5 inches tall is right below the median height of almost 63 inches. This means the person is shorter than at least half of the people in the data set, but taller than at least one fourth of them.

Teacher Tip: From the boxplot, you do not know how the other heights in the interval are distributed, so the most you can say is that the given height is above Q1. Be sure this is clear to students.

   b. Compare the conclusions you can draw about the distribution of heights by looking at just the boxplot and the conclusions you can draw by looking at just the dot plot.

   Sample Answers: Any statement about the quartiles or median can be made from the boxplot; for example, about half of the heights are between 60 and 68 inches or half of the people were taller than about 63 inches. Just by inspection you cannot make such statements from the dot plot. You can make statements about individual points from the dot plot but not from the boxplot.

Use and connect mathematical representations: *As students complete this problem, be sure that they understand the connection between the different representations for the data. The table below highlights some strategies for helping students connect the mathematical representations.*

Strategy	***Example***
Encourage purposeful selection of representations.	*Describe the data given the dot plot.* *Describe the data given the box plot.* *When would a dot plot be better to represent the data?* *When would a box plot be more appropriate to*

	represent the data?
Engage in dialogue about explicit connections among representations.	*What does the box plot tell you that the dot plot does not?* *What does the dot plot tell you that the box plot does not?*
Alternate the direction of the connections made among representations.	*If you were given the dot plot, could you construct the box plot?* *If you were given the box plot, could you construct the dot plot?* *Why or why not?*

Facilitate meaningful mathematical discourse: *In questions 5 through 11, students observe changes in the box and dot plots as they drag points in the plots. A discussion guide is included in this activity after question 11.*

5. You can move the points in the plots.
   a. Select on the point in the dot plot representing the largest height. Note that when a point is selected, it will be shaded. Move that point, and describe what happens.

      <u>Answer:</u> The corresponding point in the boxplot appears and is shaded. When you move the point in the dot plot, the point in the boxplot moves as well.

   Teacher Tip: Be sure students undo the move by selecting the reset arrow to return to the original data and plots.

b. Grab and drag the maximum point to the right, creating an outlier. At what value does it become an outlier?

   <u>Sample Answers</u>: At approximately 80, the value breaks away from the whisker.

c. Explain how you can check mathematically to see if it is really an outlier.

   <u>Sample Answers</u>: The definition for an outlier in a boxplot is any value that is greater than the upper quartile plus 1.5 times the interquartile range or that is less than the lower quartile minus 1.5

times the interquartile range. Here, 68.1 + 1.5(68.1 – 59.9) = 68.1 + 12.3 = 80.4. So any value above 80.4 will be an outlier.

Teacher Tip: Probe student thinking about outliers by having them keep the minimum and maximum unchanged and figure out how to drag points in the dot plot so that the maximum becomes an outlier. If a data point is greater than 1.5 times the IQR, then that value is an outlier. It's easy to "make an outlier" as in Question 5, but students need to think about the 1.5 IQR rule to see that changing the IQR is another way to produce an outlier. The 1.5 IQR rule can be used as a general rule of thumb for finding outliers. Students may see other methods in later coursework. This should be discussed with the class.

6. Return to the original plot by selecting the up arrow.

   a. Select the points in the dot plot representing the three largest heights. Grab and drag one point. Describe what happens in the two plots.

      Sample Answers: The corresponding points in the boxplot appear and are shaded. When you move one of the points, the entire set of selected points moves as a group in both plots**.**

   b. Select the upper whisker in the boxplot. Move one of the points in that interval, and describe what happens in the two plots.

      Sample Answers: All of the points in the upper whisker appear and are shaded. When you move one of the shaded points in either of the plots, all of the points in both plots move as a group.

7. Return to the original plot. Keep the median unchanged. Grab and drag points in the dot plot to create a symmetric boxplot.

   a. Describe your boxplot.

      Sample Answers: The median line should be more centrally located within the box. Whiskers should be approximately the same length.

   b. What did you do to get a symmetric distribution, and why?

      Sample Answers: Move data points from the right tail closer to the median line. Some points between the median and Q3 might need to be moved as well. Since each interval represents the same number of points, the goal was to equalize the spread within corresponding intervals.

TI-Nspire™ Navigator™ Opportunity: *Class Capture*

See Note 1 at the end of this lesson.

Teacher Tip: Students might want to move all of the points to the left, but since the minimum is required to stay fixed, that strategy will not work in this problem.

8. Keep the same minimum and maximum values. Grab and drag points in the dot plot to create a left-skewed distribution.

   a. Sketch your boxplot below, and describe how your process was different from the process you used in Question 7. If all you had were the boxplot, how would you describe the heights of the class?

      Sample Answers: More points are moved from left to right, and most of the points were moved from the middle regions.

   b. How many data values would you expect to find in the interval covered by the box? Explain your reasoning, and then check your answer.

      Sample Answers: There are 28 data values all together, and half of them, or 14, should be in the box part of the plot. The actual number of values might be different if some of the data values are equal.

Teacher Tip: Students might want to move all of the points to the left, but since the minimum is required to stay fixed, that strategy will not work in this problem.

9. Keep the minimum, Q1, Q3, and maximum unchanged. Describe how you would grab and drag points so the median line is closer to Q3 than to Q1. Check your answer using the two plots.

   Sample Answers: Strategies will probably differ. One might be to move two or three points from the 2nd interval (between the lower quartile and the median) into the 3rd interval. The median is the average of the 14th and 15th data points, so the strategy should be based on moving those points to the right.

Teacher Tip: Since the median is the only member of the five-number summary that is permitted to change, students will discover that points near the other quartiles should not be moved.

10. a. Describe how you would move points to change the graph so that the left whisker gets absorbed into the box.

Sample Answers: One strategy is to stack the bottom seven data points at quartile one. Answers will vary, but the whisker is defined by the lower 25% of the data points. Therefore, eliminate the distance between the 1st and 8th point.

b. Write a sentence about the distribution that you can surmise from the boxplot but not from the dot plot.

Sample Answers: Answers will vary depending on how the data points are arranged. The answers, however, should only refer to some description of the intervals and the percents they represent.

c. Write a sentence about the distribution that you can surmise from the dot plot but not from the boxplot.

Sample Answers: Answers will vary depending on how the data points are arranged but could mention gaps or clusters that cannot be seen from just looking at intervals represented by the quartiles.

11. If you observe an interval in a boxplot that is noticeably shorter than the other intervals, what does this indicate about the corresponding region in a histogram? Explain why your dot plots are reasonable.

Sample Answers: There is less variability in this interval, so the histogram would be taller. About the same number of points (depending on multiple points, etc.) are packed more closely together, so the interval that contains the points is shorter.

Facilitate meaningful mathematical discourse. *It is important to allow your students to discuss this problem to give them an opportunity to analyze and compare the ideas of others. As you listen to students work together, think about how you want to sequence students' answers for explanation to the whole class. The order in which these questions are posed is important to helping students build their understanding of the concepts. Be sure that students have time to internalize each response before moving on to the next point.*

Practices for effective discourse	***Teacher response/ actions***
Anticipating student responses	*Prior to the lesson, think about the types of answers your students may give and how you may respond. It is easier to approach the lesson with a plan for the discussion than attempting to do it on the spot. Think about common ways*

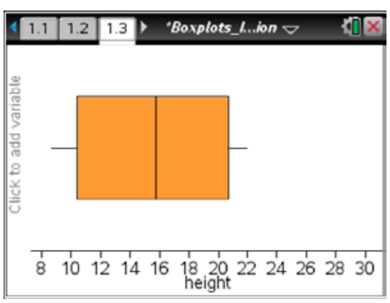
	that students are mistaken when studying these concepts and have questions ready to pose that address the misconceptions.
Monitoring student work	*As students work through the problem, make some observations about student work. Rather than providing a direct path to an answer, allow students time to reason on their own or to discuss possibilities with another student. If a student is stuck, you may ask questions to help them get started.*
Selecting students to present work	*As you observe students' work, identify solutions that can be presented to the class or group. Ask students to present their ideas in the following order.*
Sequencing students' responses	*Select student solutions to help build ideas by asking these questions.* 1. *As you moved points in the dot plot, describe what happened to the box plot.* 2. *How did you create an outlier? What makes a data point an outlier?* 3. *Describe distributions that are symmetric. What do you know about the median?* 4. *How can you move points in your dot plot so that the median does not change?*
Connecting students' responses to key ideas	• *Without moving either the maximum or minimum points, describe how you can move the fewest points in the box plot in order to create an outlier.* • *How could you create different symmetric dot plots that would result in the same boxplot?* • *How could one box plot represent data from two different data sets?* • *What changes can you make to the dot plot so that the median is closer to Q3 than it is to Q1?* • *Is it possible to have the lower quartile the same as the median? Explain why or why not.*

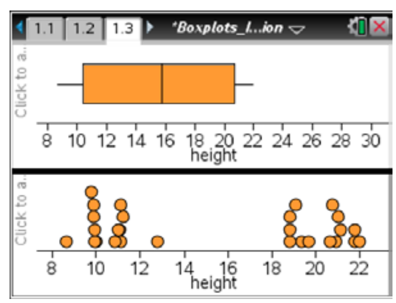
	• *Pat says that the wider the Interquartile Range (IQR), the more data are in the interval determined by the IQR. Do you agree or disagree with Pat? Support your reasoning with an example.*

Build procedural fluency from conceptual understanding. *By looking at data or a dot plot, students should understand how to find the 5-number summary values in order to construct the boxplot. The following questions help students to formulate how a dot plot may look given a box plot. In the previous discussion, students should have arrived at the idea that one box plot could represent data from two different sets, as long as the extreme values, median and quartiles were the same.*

Allow students to explore the following questions in small groups so that they can discuss the possibilities. Encourage students to construct identical boxplots for dot plots that are not evenly distributed.

12. Suppose you had additional pages in your document with the following boxplots. Sketch a possible dot plot for each and describe the shape, center, and spread. Identify a set of data you think is represented by the boxplot or dot plot.

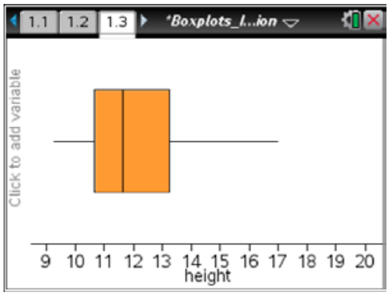
a. 

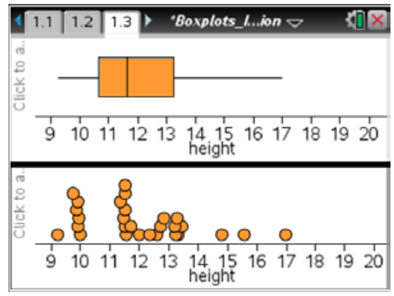


Possible answer a

Sample Answers: Center around 16, symmetric, bimodal, range appears to be 14. Identification of data will vary. Possible answers might be heights of dogs in inches, or lengths of picture frames, or any collection that might fit the descriptions summarized above.

Teacher Tip: Students' dot plots might not have gaps but should have a center lower than the tails.

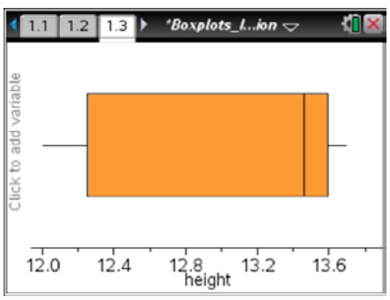
b. 

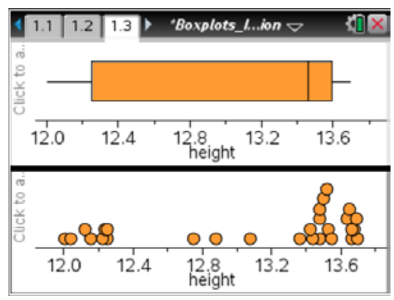


Possible answer b

<u>Sample Answers</u>: Center around 11, right skewed, unimodal, IQR about 3. Identification of data will vary.

Teacher Tip: IQR is a more appropriate measure of spread, since this distribution is unimodal.

c. 



Possible answer c

<u>Sample Answers</u>: Median is 13.548, bimodal, non-symmetric. Since the median is so close to the max, 50% of the data are near 13.5. Another 25% are around 12.25. Identification of data will vary.

Teacher Tip: You might also have students describe what possible histograms might be for each, getting students to notice that “tall” parts of histograms correspond to “short” intervals in boxplots.

Wrap Up

Upon completion of the discussion, the teacher should ensure that students are able to understand:

- Moving values across quartiles can change the five-number summary.
- Each interval of a boxplot contains approximately 25% of all the data.

- Lengths of intervals in boxplots are inversely related to the density of the data in those intervals.

TI-Nspire™ Navigator™ System

Note 1

Question 3a, *Quick Poll*

This is a good point to send a quick poll to students about the different measures.

- Which is an acceptable measure of spread of the data: mean, median, or range?
- Which is a better measure of center in this case: mean, median, mode, or range?

Note 2

Question 6, *Class Capture*

You might want to collect some class captures to ensure students are following directions and have correctly found a symmetric distribution. You might also want to have students show their work and discuss their answers to question 6b.