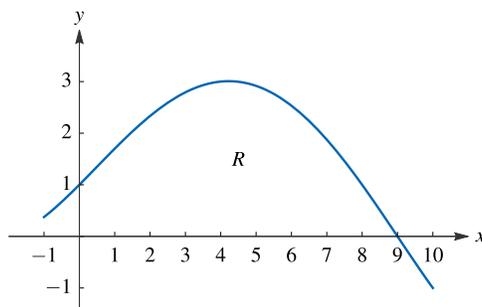


AP Calculus Mock Exam

BC 1

The graph of g' , the derivative of the twice-differentiable function g , is shown for $-1 < x < 10$. The graph of g' has exactly one horizontal tangent line, at $x = 4.2$.



Graph of g'

Let R be the region in the first quadrant bounded by the graph of g' and the x -axis from $x = 0$ to $x = 9$. It is known that $g(0) = -7$, $g(9) = 12$, and $\int_0^9 g(x) dx = 27.6$.

- Find all values of x in the interval $-1 < x < 10$, if any, at which g has a critical point. Classify each critical point as the location of a relative minimum, relative maximum, or neither. Justify your answers.
- How many points of inflection does the graph of g have on the interval $-1 < x < 10$? Give a reason for your answer.
- Find the area of the region R .
- Write an expression that represents the perimeter of the region R . Do not evaluate this expression.
- Must there exist a value of c , for $0 < c < 9$, such that $g(c) = 0$? Justify your answer.
- Evaluate $\int_0^9 \left[\frac{1}{2}g(x) - \sqrt{x} \right] dx$. Show the computations that lead to your answer.
- Evaluate $\lim_{x \rightarrow 0} \frac{x \cos x}{g(x) + 2x - 1}$. Show the computations that lead to your answer.
- Let h be the function defined by $h(x) = \int_{x^2}^0 g(t) dt$. Find $h'(3)$. Show the computations that lead to your answer.
- The region R is the base of a solid. For this solid, at each x the cross section perpendicular to the x -axis is a right triangle with height x and base in the region R . The volume of the solid is given by $\int_0^9 A(x) dx$. Write an expression for $A(x)$.
- Find the volume of the solid described in part (h). Show the computations that lead to your answer.
- Find the value of $\int_0^9 \frac{g''(x)}{g'(x)} dx$ or show that it does not exist.
- If $g''(0) = 0.7$, find the second degree Taylor polynomial for g about $x = 0$.

Solution	Scoring
<p>(a) $g'(x) = 0: x = 9$ $g'(x)$ DNE: none g has a critical point at $x = 9$. At $x = 9$, g has a relative maximum because $g'(x)$ changes from positive to negative there.</p>	<p>2: $\begin{cases} 1 : \text{critical point at } x = 9 \\ 1 : \text{relative maximum with justification} \end{cases}$</p>
<p>(b) The graph of g has a point of inflection where g' changes from increasing to decreasing or from decreasing to increasing. g' changes from increasing to decreasing at $x = 4.2$. Therefore the graph of g has one point of inflection at the point where $x = 4.2$.</p>	<p>2: $\begin{cases} 1 : \text{answer} \\ 1 : \text{reason} \end{cases}$</p>
<p>(c) Area = $\int_0^9 g'(x) dx = [g(x)]_0^9$ $= g(9) - g(0) = 12 - (-7) = 19$</p>	<p>3: $\begin{cases} 1 : \text{definite integral for area} \\ 1 : \text{Fundamental Theorem of Calculus} \\ 1 : \text{answer} \end{cases}$</p>
<p>(d) $P = 1 + 9 + \int_0^9 \sqrt{1 + g''(x)^2} dx$</p>	<p>2: $\begin{cases} 1 : \text{definite integral} \\ 1 : \text{answer} \end{cases}$</p>
<p>(e) Since g is differentiable, then g is continuous on $0 \leq x \leq 9$. $g(0) = -7 < 0 < 12 = g(9)$ By the Intermediate Value Theorem, there exists a value of c, for $0 < c < 9$, such that $g(c) = 0$.</p>	<p>2: $\begin{cases} 1 : \text{conditions} \\ 1 : \text{conclusion using the Intermediate Value Theorem} \end{cases}$</p>
<p>(f) $\int_0^9 \left[\frac{1}{2}g(x) - \sqrt{x} \right] dx = \frac{1}{2} \int_0^9 g(x) dx - \int_0^9 \sqrt{x} dx$ $= \frac{1}{2}(27.6) - \left[\frac{2}{3}x^{3/2} \right]_0^9$ $= 13.8 - \frac{2}{3}(27)$ $= 13.8 - 18 = -4.2$</p>	<p>3: $\begin{cases} 1 : \text{properties of definite integrals} \\ 1 : \text{antiderivative of } \sqrt{x} \\ 1 : \text{answer} \end{cases}$</p>

Solution	Scoring
<p>(g) $\lim_{x \rightarrow 0} (x \cos x) = 0$ $\lim_{x \rightarrow 0} (g(x) + 2x - 1) = 0$</p> <p>Therefore the limit $\lim_{x \rightarrow 0} \frac{x \cos x}{g(x) + 2x - 1}$ is in the indeterminate form $\frac{0}{0}$ and L'Hospital's Rule can be applied.</p> $\lim_{x \rightarrow 0} \frac{x \cos x}{g(x) + 2x - 1} = \lim_{x \rightarrow 0} \frac{x \cdot (-\sin x) + 1 \cdot \cos x}{g'(x) + 2}$ $= \frac{0 \cdot (-\sin 0) + 1 \cdot \cos 0}{g'(0) + 2} = \frac{1}{3}$	$3 : \begin{cases} 1 : \text{conditions for L'Hospital's Rule} \\ 1 : \text{applies L'Hospital's Rule} \\ 1 : \text{answer} \end{cases}$
<p>(h) $h'(x) = \frac{d}{dx} \left[\int_{x^2}^0 g(t) dx \right]$</p> $= -\frac{d}{dx} \left[\int_0^{x^2} g(t) dt \right]$ $= -g(x^2) \cdot (2x) = -2xg(x^2)$ $h'(3) = -2 \cdot 3 \cdot g(9) = -6 \cdot 12 = -72$	$3 : \begin{cases} 1 : \text{Fundamental Theorem of Calculus} \\ 1 : \text{Chain Rule} \\ 1 : \text{answer} \end{cases}$
<p>(i) $A(x)$ represents the area of a right triangle at each x.</p> $A(x) = \frac{1}{2} x g'(x)$	<p>1 : answer</p>
<p>(j) $V = \int_0^9 A(x) dx = \frac{1}{2} \int_0^9 x g'(x) dx$</p> <p>Use integration by parts.</p> $u = x \quad dv = g'(x) dx$ $du = dx \quad v = \int g'(x) dx = g(x)$ $V = \frac{1}{2} \left([x \cdot g(x)]_0^9 - \int_0^9 g(x) dx \right)$ $= \frac{1}{2} ([9 \cdot g(9) - 0 \cdot g(0)] - 27.6)$ $= \frac{1}{2} (9 \cdot 12 - 27.6) = 40.2$	$2 : \begin{cases} 1 : \text{integration by parts} \\ 1 : \text{answer} \end{cases}$

Solution	Scoring
<p>(k) $\int_0^9 \frac{g''(x)}{g'(x)} dx = \lim_{t \rightarrow 9^-} \int_0^t \frac{g''(x)}{g'(x)} dx$</p> <p>Let $u = g'(x)$, then $du = g''(x) dx$ and $dx = \frac{du}{g''(x)}$</p> $\int \frac{g''(x)}{g'(x)} dx = \int \frac{g''(x)}{u} \cdot \frac{du}{g''(x)} = \int \frac{du}{u}$ $= \ln u = \ln g'(x) $ $\lim_{t \rightarrow 9^-} \int_0^t \frac{g''(x)}{g'(x)} dx = \lim_{t \rightarrow 9^-} [\ln g'(x)]_0^t$ $= \lim_{t \rightarrow 9^-} [\ln g'(t) - \ln g'(0)]$ $= \lim_{t \rightarrow 9^-} \ln g'(t) = -\infty$ <p>Therefore the improper integral does not exist.</p>	$3 : \begin{cases} 1 : \text{improper integral} \\ 1 : \text{antiderivative} \\ 1 : \text{answer} \end{cases}$
<p>(l) $g(0) = -7, \quad g'(0) = 1, \quad g''(0) = 0.7$</p> $T_2(x) = g(0) + g'(0)x + \frac{g''(0)}{2!}x^2$ $= -7 + 1 \cdot x + \frac{0.7}{2}x^2$ $= -7 + x + 0.35x^2$	$2 : \begin{cases} 1 : \text{form of } T_2(x) \\ 1 : \text{answer} \end{cases}$