

TI in Focus: AP[®] Calculus

2018 AP[®] Calculus Exam: AB-2

Technology Solutions and Problem Extensions

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Outline

- (1) Free Response Question
- (2) Scoring Guidelines
- (3) Solutions using technology
- (4) Problem Extensions

2. A particle moves along the x -axis with velocity given by $v(t) = \frac{10 \sin(0.4t^2)}{t^2 - t + 3}$ for time $0 \leq t \leq 3.5$.

The particle is at position $x = -5$ at time $t = 0$.

- (a) Find the acceleration of the particle at time $t = 3$.
- (b) Find the position of the particle at time $t = 3$.
- (c) Evaluate $\int_0^{3.5} v(t) dt$, and evaluate $\int_0^{3.5} |v(t)| dt$. Interpret the meaning of each integral in the context of the problem.
- (d) A second particle moves along the x -axis with position given by $x_2(t) = t^2 - t$ for $0 \leq t \leq 3.5$. At what time t are the two particles moving with the same velocity?

(a) $v'(3) = -2.118$

The acceleration of the particle at time $t = 3$ is -2.118 .

1 : answer

(b) $x(3) = x(0) + \int_0^3 v(t) dt = -5 + \int_0^3 v(t) dt = -1.760213$

The position of the particle at time $t = 3$ is -1.760 .

3 : $\begin{cases} 1 : \int_0^3 v(t) dt \\ 1 : \text{uses initial condition} \\ 1 : \text{answer} \end{cases}$

(c) $\int_0^{3.5} v(t) dt = 2.844$ (or 2.843)

$$\int_0^{3.5} |v(t)| dt = 3.737$$

The integral $\int_0^{3.5} v(t) dt$ is the displacement of the particle over the time interval $0 \leq t \leq 3.5$.

The integral $\int_0^{3.5} |v(t)| dt$ is the total distance traveled by the particle over the time interval $0 \leq t \leq 3.5$.

3 : $\begin{cases} 1 : \text{answers} \\ 2 : \text{interpretations of } \int_0^{3.5} v(t) dt \\ \text{and } \int_0^{3.5} |v(t)| dt \end{cases}$

(d) $v(t) = x_2'(t)$

$$v(t) = 2t - 1 \Rightarrow t = 1.57054$$

The two particles are moving with the same velocity at time $t = 1.571$ (or 1.570).

$$2 : \begin{cases} 1 : \text{sets } v(t) = x_2'(t) \\ 1 : \text{answer} \end{cases}$$

Part (a)

$$v(t) = \frac{10 \sin(0.4t^2)}{t^2 - t + 3}$$

$$a(3) = v'(3) = -2.118$$

TI-84 Plus calculator screenshot showing the function $v(t) = \frac{10 \cdot \sin(0.4 \cdot t^2)}{t^2 - t + 3}$ and the derivative $\frac{d}{dt}(v(t))|_{t=3} = -2.1182$. The mode is set to RAD.

TI-84 Plus calculator screenshot showing the function $v(t) = \frac{10 \cdot \sin\left(\frac{2}{5} \cdot t^2\right)}{t^2 - t + 3}$ and the derivative $\frac{d}{dt}(v(t))|_{t=3} = \frac{8 \cdot \cos\left(\frac{18}{5}\right)}{3} - \frac{50 \cdot \sin\left(\frac{18}{5}\right)}{81}$. The mode is set to RAD.

Part (b)

$$x(3) = x(0) + \int_0^3 v(t) dt$$

$$= -5 + 3.32979$$

$$= -1.76$$

FTC or Net Change Theorem

Evaluate definite integral

Simplify.

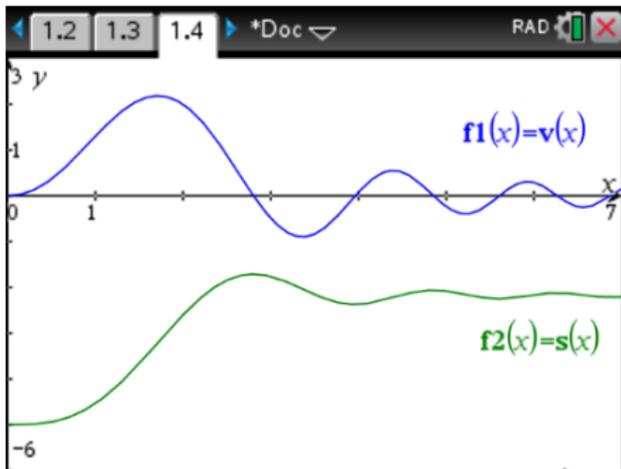
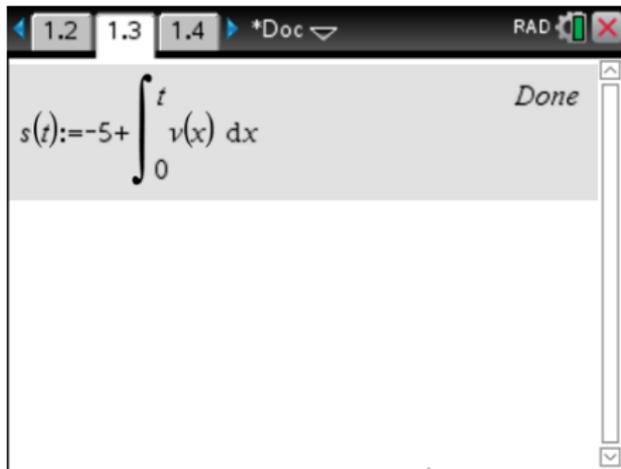
A screenshot of a TI-84 Plus calculator interface. The top status bar shows '1.1', '1.2', '2.1', '*Doc', and 'RAD'. The main display area shows the integral $\int_0^3 v(t) dt$ with the result '3.23979' to its right. Below this, the value '3.2397868128047-5' is displayed on the left and '-1.76021' is displayed on the right.

Example 1 Position Function

Suppose the particle moves along the x -axis with velocity given by $v(t)$ for time $0 \leq t \leq 7$.

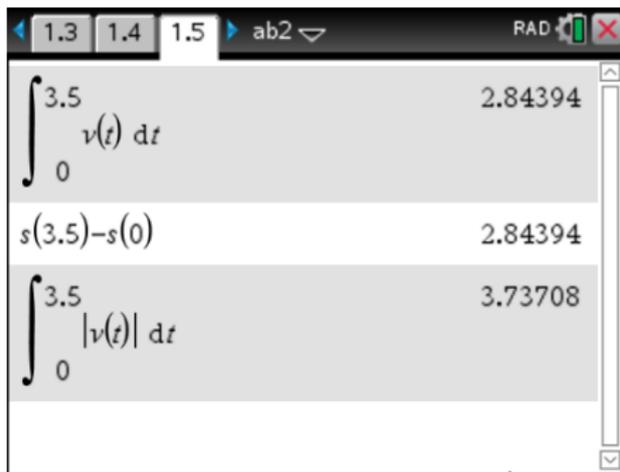
- Sketch a graph of the position function.
- At what time t is the particle farthest to the right? What is the position of the particle at that time?

Solution



Part (c)

Evaluate the two definite integrals.

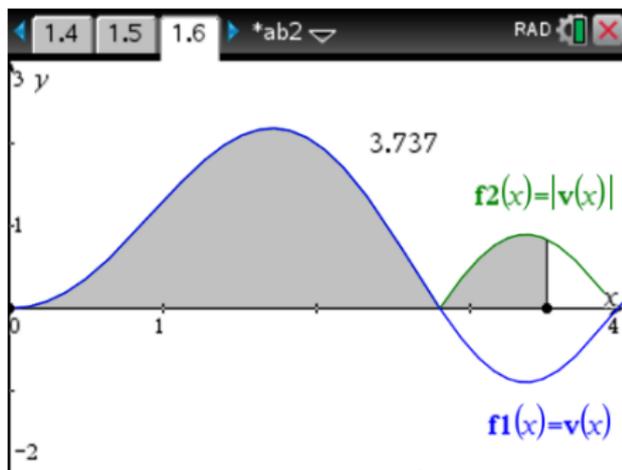
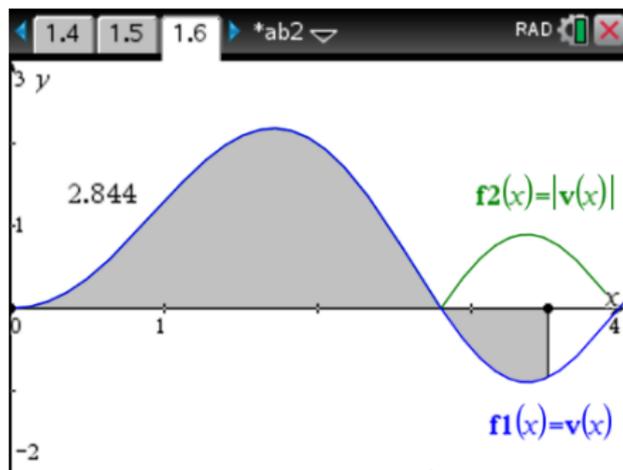


$\int_0^{3.5} v(t) dt$: displacement of the particle over the time interval $0 \leq t \leq 3.5$.

$\int_0^{3.5} |v(t)| dt$: total distance traveled by the particle over the time interval $0 \leq t \leq 3.5$.

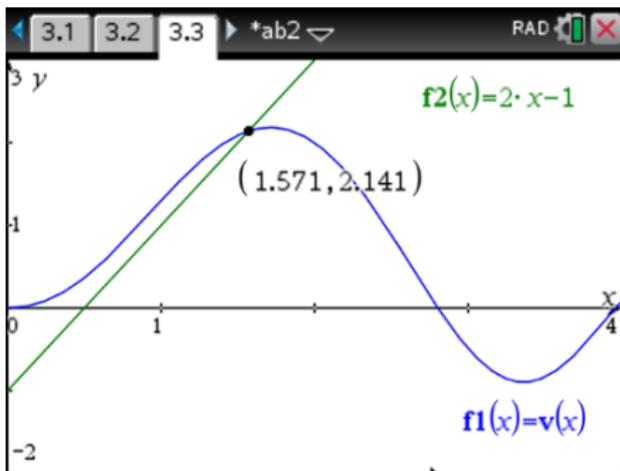
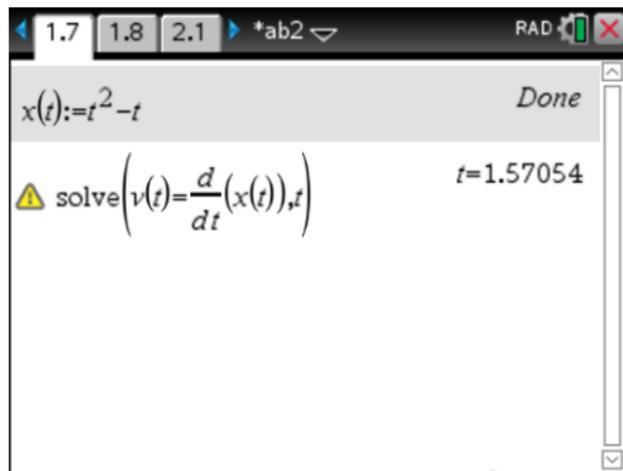
Part (c)

These graphs illustrate the two definite integrals.



Part (d)

$$v(t) = x'(t) = 2t - 1 \implies t = 1.571$$



Example 2 More Particles

Particle 1 moves along along the x -axis with velocity given by

$$v_1(t) = \frac{10 \sin(0.4t^2)}{t^2 - t + 3} \text{ for time } 0 \leq t \leq 7 \text{ and is at position } x = -5 \text{ at time } t = 0.$$

Particle 2 also moves along the x -axis with velocity given by $v_2(t) = 1.5 \cos(1.5t)$ for time $0 \leq t \leq 7$ and is at position $x = -1$ at time $t = 0$.

- Find the position of particle 2 at time $t = 5$.
- Find the total distance traveled for particle 2 over the time interval $0 \leq t \leq 7$.
- Find the times when the two particles are traveling in the same direction.
- Find the time when the two particles are farthest apart.
- Find the time(s) when the particles are at the same position and determine whether the particles are moving in the same direction or opposite directions.
- Find $\lim_{t \rightarrow \infty} v(t)$ and explain this result in the context of the problem.

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