

2017 AP Calculus Exam: AB-5

Technology Uses and Problem Extensions

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AB-5

5. Two particles move along the x -axis. For $0 \leq t \leq 8$, the position of particle P at time t is given by

$$x_P(t) = \ln(t^2 - 2t + 10),$$
 while the velocity of particle Q at time t is given by $v_Q(t) = t^2 - 8t + 15$.

Particle Q is at position $x = 5$ at time $t = 0$.

- For $0 \leq t \leq 8$, when is particle P moving to the left?
- For $0 \leq t \leq 8$, find all times t during which the two particles travel in the same direction.
- Find the acceleration of particle Q at time $t = 2$. Is the speed of particle Q increasing, decreasing, or neither at time $t = 2$? Explain your reasoning.
- Find the position of particle Q the first time it changes direction.

AB-5

$$(a) \quad x'_p(t) = \frac{2t-2}{t^2-2t+10} = \frac{2(t-1)}{t^2-2t+10}$$

$$t^2 - 2t + 10 > 0 \text{ for all } t.$$

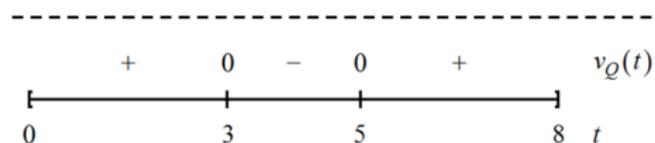
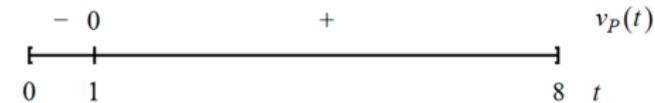
$$x'_p(t) = 0 \Rightarrow t = 1$$

$$x'_p(t) < 0 \text{ for } 0 \leq t < 1.$$

Therefore, the particle is moving to the left for $0 \leq t < 1$.

$$(b) \quad v_Q(t) = (t-5)(t-3)$$

$$v_Q(t) = 0 \Rightarrow t = 3, t = 5$$



Both particles move in the same direction for $1 < t < 3$ and $5 < t \leq 8$ since $v_p(t) = x'_p(t)$ and $v_Q(t)$ have the same sign on these intervals.

$$2: \begin{cases} 1: x'_p(t) \\ 1: \text{interval} \end{cases}$$

$$2: \begin{cases} 1: \text{intervals} \\ 1: \text{analysis using } v_p(t) \text{ and } v_Q(t) \end{cases}$$

Note: 1/2 if only one interval with analysis

Note: 0/2 if no analysis

$$\begin{aligned} \text{(c)} \quad a_Q(t) &= v_Q'(t) = 2t - 8 \\ a_Q(2) &= 2 \cdot 2 - 8 = -4 \end{aligned}$$

$$a_Q(2) < 0 \text{ and } v_Q(2) = 3 > 0$$

At time $t = 2$, the speed of the particle is decreasing because velocity and acceleration have opposite signs.

(d) Particle Q first changes direction at time $t = 3$.

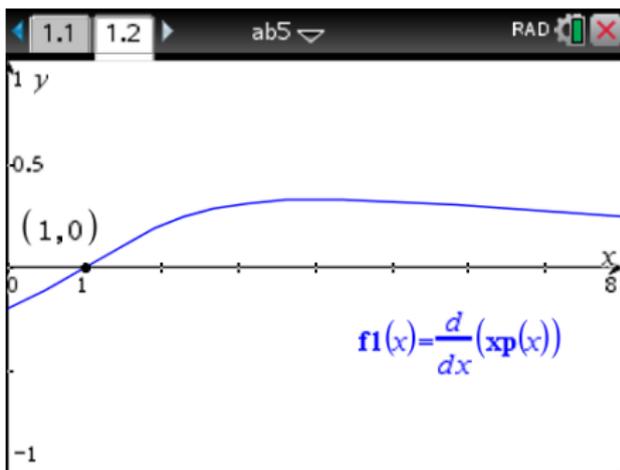
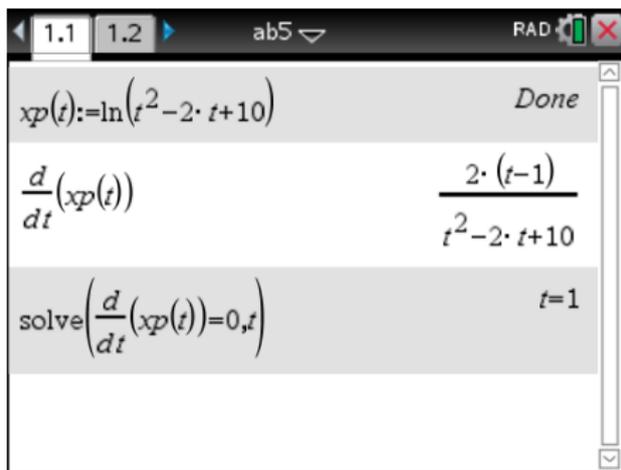
$$\begin{aligned} x_Q(3) &= x_Q(0) + \int_0^3 v_Q(t) dt = 5 + \int_0^3 (t^2 - 8t + 15) dt \\ &= 5 + \left[\frac{1}{3}t^3 - 4t^2 + 15t \right]_{t=0}^{t=3} = 5 + (9 - 36 + 45) = 23 \end{aligned}$$

$$2 : \begin{cases} 1 : a_Q(2) \\ 1 : \text{speed decreasing with reason} \end{cases}$$

$$3 : \begin{cases} 1 : \text{antiderivative} \\ 1 : \text{uses initial condition} \\ 1 : \text{answer} \end{cases}$$

Part (a)

Technology Solution



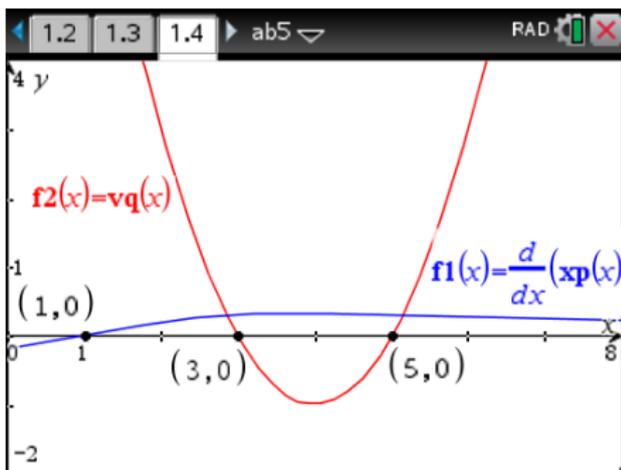
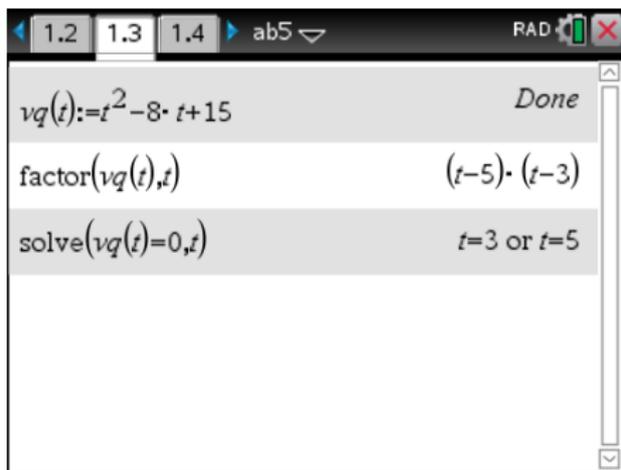
$$x'_P(t) = 0 \implies t = 1$$

$$x'_P(t) < 0 \text{ for } 0 \leq t < 1$$

Therefore, the particle is moving to the left for $0 \leq t < 1$.

Part (b)

Technology Solution



Both particles are moving in the same direction for $1 < t < 3$ and $5 < t \leq 8$ since $v_p(t) = x'_p(t)$ and $v_q(t)$ have the same sign on these intervals.

Part (c)

Technology Solution

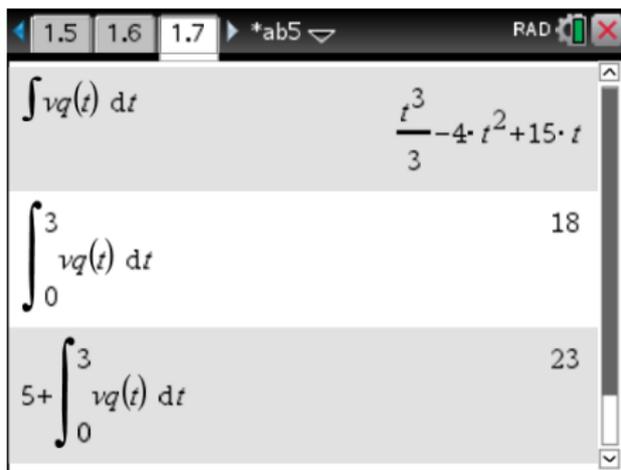
A screenshot of a TI-84 Plus calculator interface. The top status bar shows the mode set to 'RAD' and the cursor on the '1.5' menu item. The main display area shows three rows of results:

$\frac{d}{dt}(vq(t))$	$2 \cdot t - 8$
$\frac{d}{dt}(vq(t)) _{t=2}$	-4
$vq(2)$	3

At time $t = 2$, the speed of the particle is decreasing because velocity and acceleration have opposite signs.

Part (d)

Technology Solution



Particle Q first changes direction at time $t = 3$.

The position of the particle at time $t = 3$: 23

Problem Extensions

(1) Find the average acceleration for particle Q over the interval $[0, 8]$.

Solution

$$\begin{aligned} a_{\text{ave}} &= \frac{1}{8} \int_0^8 x''_P(t) dt \\ &= \frac{1}{8} [x'_P(t)]_0^8 \\ &= \frac{1}{8} \left[\frac{7}{29} - \left(-\frac{1}{5} \right) \right] \\ &= \frac{8}{145} \end{aligned}$$

The image shows a TI-84 Plus calculator screen with the following content:

- Top row: Navigation buttons (left arrow, 1.6, 1.7, 1.8, right arrow) and a dropdown menu showing '*ab5'.
- Top right: 'RAD' indicator and a close button (X).
- Input line: $v_P(t) := \frac{d}{dt}(x_P(t))$ followed by a 'Done' button.
- Second line: A list command $\{v_P(8), v_P(0)\}$ resulting in the list $\left\{ \frac{7}{29}, -\frac{1}{5} \right\}$.
- Third line: A fraction calculation $\frac{1}{8} \cdot (v_P(8) - v_P(0))$ resulting in the fraction $\frac{8}{145}$.

Problem Extensions

(2) Find the speed of particle P and the speed of particle Q at time $t = 4$.

Solution

$$|x'_P(4)| = \left| \frac{2(4-2)}{4^2 - 2 \cdot 4 + 10} \right| = \frac{1}{3}$$

$$|v_Q(4)| = |4^2 - 8 \cdot 4 + 15| = |-1| = 1$$

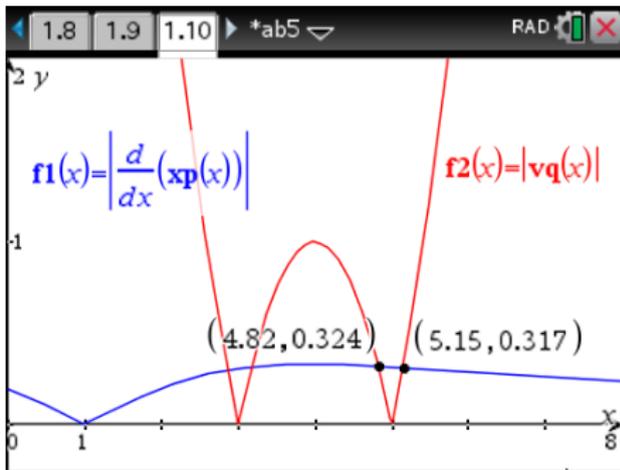
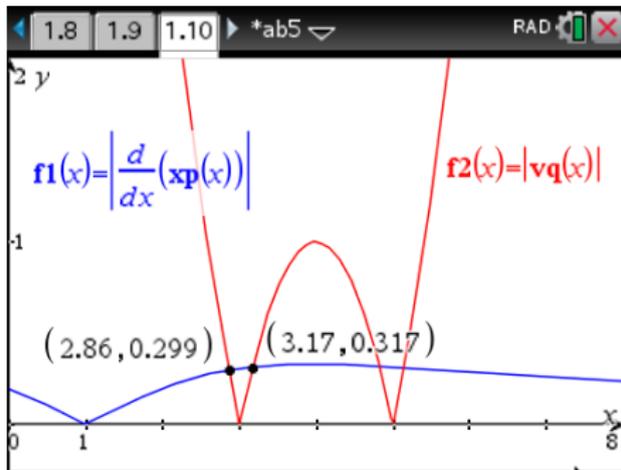
A screenshot of a TI-84 Plus calculator interface. The top status bar shows the mode set to 'RAD' and the angle indicator set to 'ab5'. The calculator screen displays two results:

$\left \frac{d}{dt}(x_P(t)) \right _{t=4}$	$\frac{1}{3}$
$ v_Q(4) $	1

Problem Extensions

- (3) For $0 \leq t \leq 8$, find all times t during which the speed of particle P is greater than particle Q .

Solution



The speed of particle P is greater than the speed of particle Q for $(2.86, 3.17)$ and $(4.82, 5.15)$.

Problem Extensions

(4) Find the total distance traveled for particle P over the interval $[0, 8]$.

Solution

$$\begin{aligned} & \int_0^8 |x'_P(t)| dt \\ &= \int_0^1 -x'_P(t) dt + \int_1^8 x'_P(t) dt \\ &= -[x_P(t)]_0^1 + [x_P(t)]_1^8 \\ &= -[\ln 9 - \ln 10] + [\ln 58 - \ln 9] \\ &= \ln \frac{580}{51} \end{aligned}$$

A screenshot of a TI-84 Plus calculator interface. The top status bar shows '1.9', '1.10', '1.11', '*ab5', and 'RAD'. The main display shows the integral expression $\int_0^8 \left| \frac{d}{dt}(x_P(t)) \right| dt$ and the result $\ln\left(\frac{580}{51}\right)$.

Problem Extensions

- (5) Graph the position, velocity, and acceleration functions for the particle P over the interval $[0, 8]$. (Explain the relationships among the three graphs.)

Solution

$$x_P(t) = \ln(t^2 - 2t + 10)$$

$$v_P(t) = x'_P(t) = \frac{2t - 2}{t^2 - 2t + 10} = \frac{2(t - 1)}{t^2 - 2t + 10}$$

$$\begin{aligned} a_P(t) &= \frac{(t^2 - 2t + 10)(2) - 2(t - 1)(2t - 2)}{(t^2 - 2t + 10)^2} \\ &= \frac{-2t^2 - 4t + 20 - 4t^2 + 8t - 4}{(t^2 - 2t + 10)^2} \\ &= \frac{-2(t^2 - 2t + 8)}{(t^2 - 2t + 10)^2} \end{aligned}$$

Problem Extensions

Solution

1.10 1.11 2.1 *ab5 ▾ RAD  

$xp(t) := \ln(t^2 - 2 \cdot t + 10)$ Done

$vp(t) := \frac{d}{dt}(xp(t))$ Done

$vp(t)$ $\frac{2 \cdot (t-1)}{t^2 - 2 \cdot t + 10}$

1.11 2.1 2.2 *ab5 ▾ RAD  

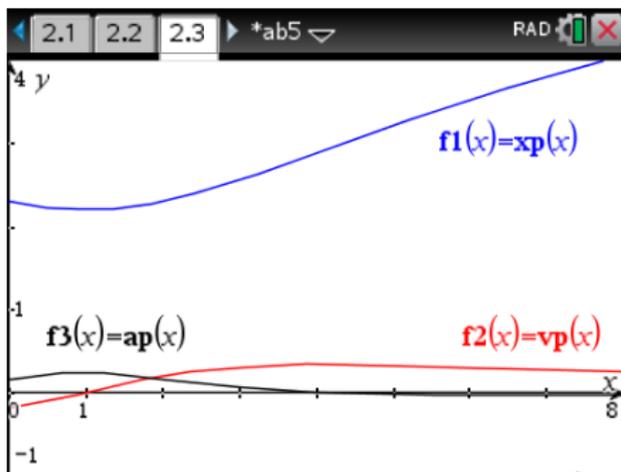
$ap(t) := \frac{d}{dt}(vp(t))$ Done

$ap(t)$ $\frac{-2 \cdot (t^2 - 2 \cdot t - 8)}{(t^2 - 2 \cdot t + 10)^2}$



Problem Extensions

Solution



The position function is increasing where the velocity is positive.

The position function is decreasing where the velocity is negative.