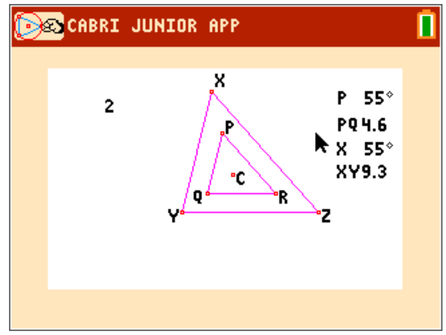
Constructing Similar Triangles



About the Lesson

In this activity, students will investigate three different methods of constructing similar triangles. They will use the **Dilation** tool with the dilation point inside and outside of the triangle to investigate different relationships. Also, students will use the **Parallel** tool to construct two similar triangles from one triangle. As a result, students will:

- Use inductive reasoning to classify each set of conditions as necessary and/or sufficient for similarity:
  - The side lengths of one triangle are equal to the corresponding side lengths of another triangle.
  - Two angles of one triangle are congruent to two angles of another triangle.

Vocabulary

- similar
- congruent
- dilation
- scale factor

Teacher Preparation and Notes

- Before starting this activity, students should be familiar with the term dilation.
- These lesson notes include step-by-step directions for the constructions needed for this lesson. If students are familiar with the constructions in Cabri Jr., they can work through the activity in small groups.

Activity Materials

- Compatible TI Technologies:

  TI-84 Plus*
  TI-84 Plus Silver Edition*
  TI-84 Plus C Silver Edition
  TI-84 Plus CE

** with the latest operating system (2.55MP) featuring MathPrint ™ functionality.*

Tech Tips:

- This activity includes screen captures taken from the TI-84 Plus CE. It is also appropriate for use with the rest of the TI-84 Plus family. Slight variations to these directions may be required if using other calculator models.
- Watch for additional Tech Tips throughout the activity for the specific technology you are using.
- Access free tutorials at http://education.ti.com/calculators/pd/US/Online-Learning/Tutorials
- Any required calculator files can be distributed to students via handheld-to-handheld transfer.

Lesson Files:

- Constructing_Similar_Triangles_Student.pdf
- Constructing_Similar_Triangles_Student.doc

Students need to press [apps] and select **CabriJr** to start the application. When they open a new document, they need to make sure that the axes are hidden. If the axes are displayed, press [graph] **> Hide/Show > Axes**.

Explain to students that similar triangles are those that have the same shape but not necessarily the same size.

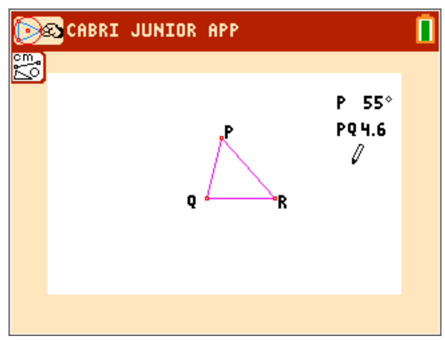
Congruent triangles are a special type of similar triangle where corresponding sides are congruent. In similar triangles, corresponding angles are congruent but corresponding sides are proportional. In this activity, students will look at three methods of constructing similar triangles and will test these properties using dilations or stretches.

In order to examine all of the sides and angles, students should work in groups of three. Have one student (Student A on the worksheet) in each group construct the first triangle and save it as "**SIMTRI**." This is saved in the TI-84 Plus family as an **APPVAR**. Student A needs to transfer the **APPVAR** to the other two students in the group.

Problem 1 – Similar Triangles Using Dilation

Student A will use the **Triangle** tool to construct a triangle of any size. Then they need to use the **Alph-Num** tool to label the vertices *P*, *Q*, and *R*.

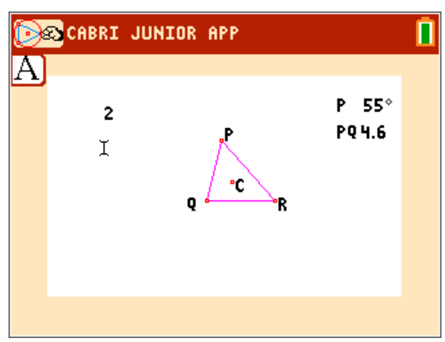
Student A should measure $\angle P$ and $\overline{PQ}$, Student B should measure $\angle Q$ and side $\overline{QR}$, Student C should measure $\angle R$ and $\overline{PR}$.



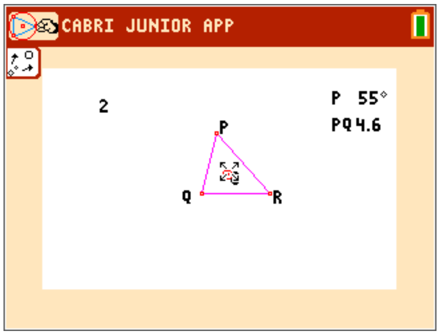
Note: To increase the number of digits for the length of the side, hover the cursor over the measurement and press [+].

With the **Point** tool, students on their own calculators will construct a point *C* in the center of the triangle. They will then use the **Alph-Num** tool to place the number **2** at the top of the screen.

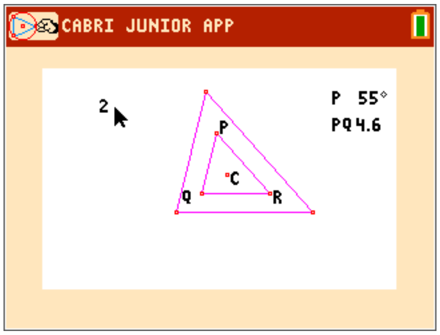
Explain to students that point *C* will be the center of the dilation and number **2** will be the scale factor.



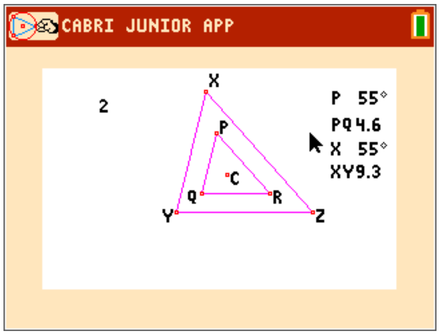
With the **Dilation** tool selected, students need to move the cursor to point *C*, the center of the dilation, and press enter. They should notice that the shape of the cursor changes. This is to indicate that everything will be stretched away from this point. Then students move the cursor to the perimeter of the triangle and press enter. Finally, move to the scale factor, **2**, and press enter.



Students will see that a new, larger triangle will appear outside of △*PQR*. Have a preliminary discussion with students about whether they think the new triangle is similar to △*PQR* and how they can confirm their hypothesis.



Instruct students to label this triangle as *XYZ* so that *X* corresponds to *P*, *Y* to *Q*, and *Z* to *R*. Each person in the group should select and measure their appropriate angle and side in the new triangle. Students should answer Questions 1 and 2 on the worksheet comparing the two angles and two sides.



1. What do you notice about the two angles? Compare this to the other students in your group.

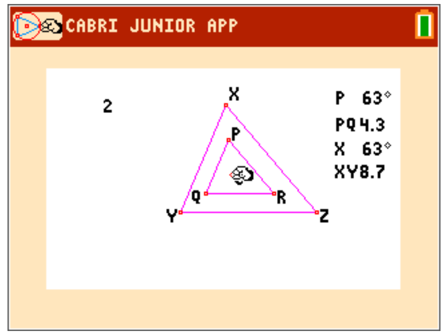
Sample Answer: One triangle fits inside of the other. Each side of one triangle is parallel to a side of the other triangle.

2. How do the lengths of the sides compare? Is this the result that you were expecting?

Sample Answer: The length of the sides of the bigger triangle are twice the length of the sides of the smaller triangle.

Explain to students that $\overline{XY} = 2(\overline{PQ})$ indicates that the sides have the ratio 2:1. If all three sides display the same result, then the sides are said to be proportional.

To answer Questions 3 and 4 on the worksheet, students will observe the changes in the triangles as they drag a point in the original triangle and the point of dilation.



3. Predict what will happen to the corresponding angles and sides when a point on $\triangle PQR$ is moved. Drag your point in $\triangle PQR$. Do the corresponding angles remain congruent? Does the relationship between corresponding sides remain the same? Compare your results to others in your group.

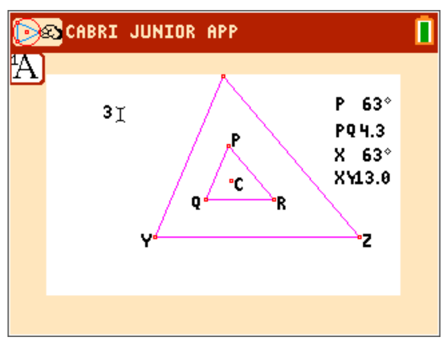
<u>**Sample Answer:**</u> I think that the angles will remain congruent and the relationship between the corresponding sides will remain the same. After moving a point, my predictions were true.

4. Drag point *C*. Are the relationships preserved under this change? Compare your results to others in your group. Does it make any difference that each person may have constructed a different center point.

<u>**Sample Answer:**</u> The relationships are still the same. The center point does not make a difference in the relationship.

Problem 2 – Different Scale Factors

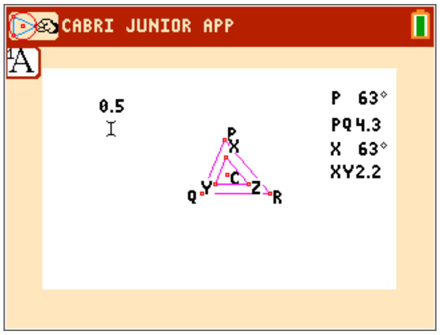
Students will continue using the same file. To change the scale factor, students must select the **Alph-Num** tool, move the cursor to the scale factor of 2, and press [enter]. Then they should delete **2** by pressing [del], press [alpha] to change the character to a number, and enter **3**. Students can now answer Question 5 on the worksheet.



5. What happens to your construction? Does this change the relationships you found in Problem 1?

<u>**Sample Answer:**</u> The lengths of the corresponding sides of $\triangle XYZ$ are now 3 times the lengths of $\triangle PQR$.

Now students are to investigate another scale factor by changing it from 3 to 0.5. They should then answer Questions 6 and 7 on the worksheet, summarizing their observations of the triangles and their measurements.



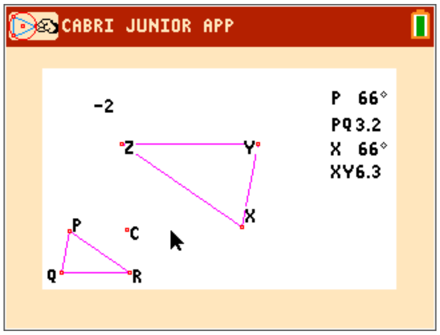
6. Change the scale factor from 3 to 0.5. How does this affect your construction?

<u>Sample Answer:</u> The lengths of the corresponding sides of $\triangle XYZ$ are now half of the lengths of $\triangle PQR$.

7. Summarize your findings by stating the effect of a dilation on corresponding angles and sides.

<u>Sample Answer:</u> The lengths of the corresponding sides of $\triangle XYZ$ will always be the lengths of $\triangle PQR$ times the scale factor. The angles will always remain the same.

For the last investigation in this problem, students will look at the effect of a dilation when the center point *C* is outside of the pre-image triangle and a negative scale factor is used. Students will need to move $\triangle PQR$ by moving the cursor to one side and when all sides of the triangle flash, press [alpha], and then use the arrow keys.

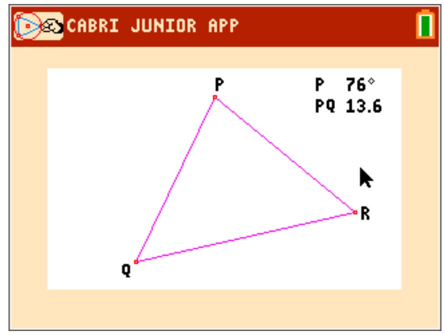


8. Drag $\triangle PQR$ to the lower left corner and drag point *C* to the right of the triangle. Change the scale factor to –2. Are the properties that you noted above preserved by these changes?

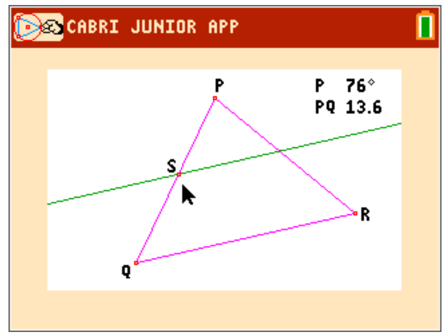
<u>Sample Answer:</u> The properties are preserved. However, $\triangle XYZ$ is now a reflection of $\triangle PQR$.

Problem 3 – Similar Triangles with a Parallel Line

Finally, students will look at a completely different method for constructing similar triangles. All students will need to open a new file. It is the teacher's decision to have them save the file. Student A should construct a $\triangle PQR$ and transfer it to the others in their group. They will all measure their same side and angle as before.

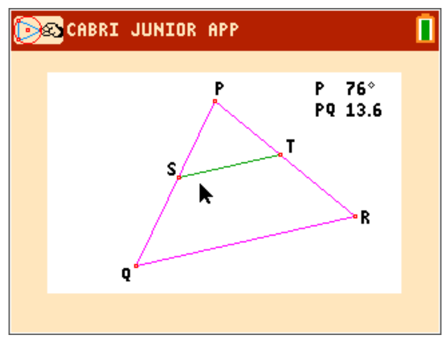


All students need to construct a point on $\overline{PQ}$ using the **Point on** tool and label it S using the **Alph-Num** tool. Using this point, they should construct a line that is parallel to $\overline{QR}$ by selecting the **Parallel** tool, and then choosing the point and side.

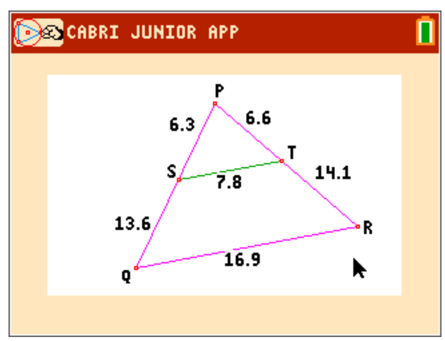


With the **Intersection** tool, students can find the intersection of $\overline{PR}$ and the parallel line, and then use the **Alph-Num** tool to label it as T.

The **Hide/Show > Object** tool enables students to hide the parallel line, and then construct $\overline{ST}$ with the **Segment** tool. If students can prove that all of the angles are congruent, they have completed their first step in proving that the two triangles are similar.

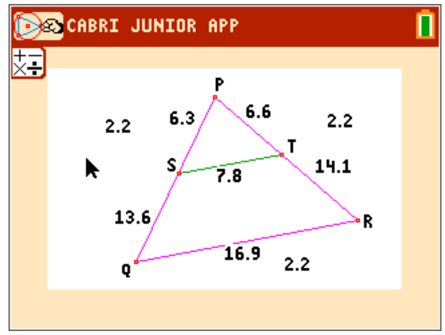


With the **Measure > D. & Length** tool, students can measure segments of the corresponding sides. In the screen to the right, all three pairs of sides have been measured.



To complete the proof, students will need to confirm that all of the sides are proportional. Using the **Calculate** tool, they can select a side measurement, press the operation key (÷), and then select the corresponding side measurement.

If all three ratios are equivalent, then the sides are proportional and the two triangles are similar.



9. Describe how you can prove whether or not all three pairs of corresponding angles are congruent. If they are congruent, then $\triangle PST$ is similar to $\triangle PQR$.

Sample Answer: I can measure all of the pairs of corresponding angles. All of the pairs of corresponding angles are congruent, so $\triangle PST$ is similar to $\triangle PQR$.

10. Calculate the ratio of PQ:PS. Then calculate the ratios of the other sides. If all the ratios are equivalent, then the sides are proportional. Are the sides in $\triangle PST$ and $\triangle PQR$ proportional?

Sample Answer: The ratios of the corresponding sides are all 2:2. The sides are proportional.