Bridge on the River Quad

About the Lesson

In the first part of this activity, students graph a quadratic function that models the shape of a bridge trestle. They then solve the related quadratic equation by completing the square, recording each step as they complete it. This list of steps is then generalized to deduce the quadratic formula. In the second part of the activity, students store the formula in their graphing calculator, compare its results with those of the Equation Solver (optional), and use it to solve several other quadratic equations. As a result, students will:

- Graph a quadratic function $y = ax^2 + bx + c$ and display a table of integral values of the variable.
- Convert a quadratic function $y = ax^2 + bx + c$ to the form $y = a(x - h)^2 + k$ by completing the square and deduce the formula for the roots of a general quadratic equation.
- Use the Equation Solver to verify the roots of a quadratic equation obtained by the quadratic formula (optional).

Vocabulary

- quadratic formula
- vertex
- intercepts

Teacher Preparation and Notes

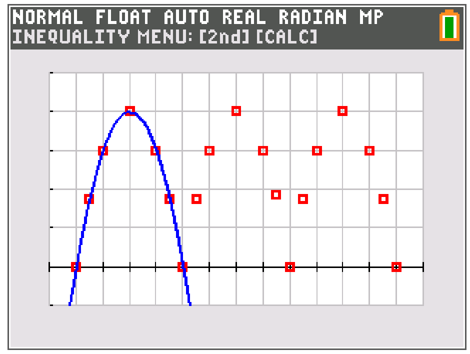
- This activity presents several methods for solving quadratic equations and is appropriate for students in Algebra 1. Prior to beginning this activity, students should be familiar with solving quadratic equations by completing the square.

Activity Materials

- Compatible TI Technologies:

  TI-84 Plus*
  TI-84 Plus Silver Edition*
  TI-84 Plus C Silver Edition
  TI-84 Plus CE

** with the latest operating system (2.55MP) featuring MathPrint™ functionality.*



Tech Tips:

- This activity includes screen captures taken from the TI-84 Plus CE. It is also appropriate for use with the rest of the TI-84 Plus family. Slight variations to these directions may be required if using other calculator models.
- Watch for additional Tech Tips throughout the activity for the specific technology you are using.
- Access free tutorials at http://education.ti.com/calculators/pd/US/Online-Learning/Tutorials
- Any required calculator files can be distributed to students via handheld-to-handheld transfer.

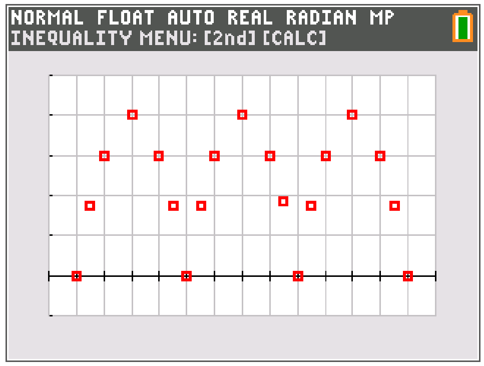
Lesson Files:

- Bridge_on_the_River_Quad_Student.pdf
- Bridge_on_the_River_Quad_Student.doc

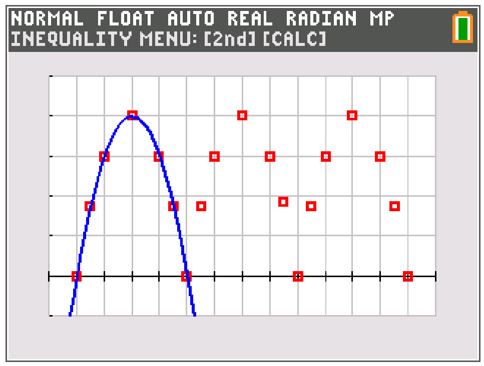
Problem 1 – Solving a Quadratic Equation by Completing the Square

Students are presented with the scenario of the problem: a bridge with four parabolic trestles. Students are given the equation $y = -x^2 + 9x - 16$ as a model of the shape of a trestle and prompted to graph it.

Ask: *How can you find the x-coordinates of the points where this parabola crosses the x-axis?*

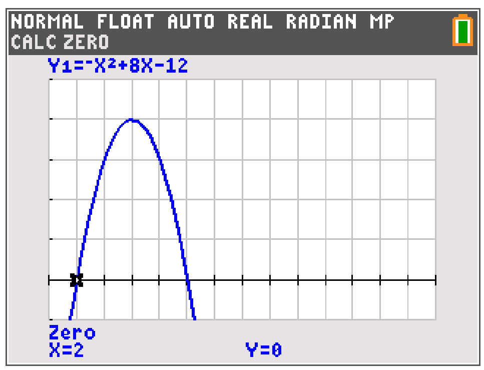


Students should realize that they can set the equation equal to 0 and solve for x. Discuss the connection between the function $f(x) = -x^2 + 9x - 16$ and the equation $-x^2 + 9x - 16 = 0$. The function describes the entire graph, while the equation is true for only the two points where the graph interests the x-axis.

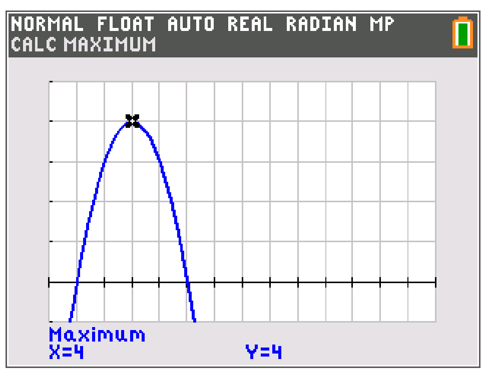


Students can work independently to complete the square for the equation $-x^2 + 9x - 16 = 0$. Remind them that their first step must be to divide both sides of the equation by -1, in order to make the coefficient of x^2 equal to 1.

The student worksheet provides a place for students to record their work as well as a description of each step. The steps are shown in the solutions.



Students should check their algebra by comparing the equation that results from completing the square with the coordinates of the vertex of the parabola. In a whole class setting, demonstrate the **Maximum** tool in the **Calc** menu. Press enter to input a left bound, a right bound, and a guess. If the square was completed correctly, the coordinates of the vertex match the values of h and k in students' equations.



Deriving the Quadratic Formula

Algebra	Step
$ax^2 + bx + c = 0$	original problem
$\frac{ax^2}{a} + \frac{bx}{a} + \frac{c}{a} = \frac{0}{a}$	divide both sides by a
$x^2 + \frac{bx}{a} + \frac{c}{a} = 0$	simplify
$x^2 + \frac{bx}{a} + \frac{c}{a} - \frac{c}{a} = 0 - \frac{c}{a}$	subtract $\frac{c}{a}$ from both sides
$x^2 + \frac{bx}{a} + \left(\frac{b}{2a}\right)^2 = -\frac{c}{a} + \left(\frac{b}{2a}\right)^2$	add $\left(\frac{b}{2a}\right)^2$ to both sides
$\left(x + \frac{b}{2a}\right)^2 = -\frac{c}{a} + \left(\frac{b}{2a}\right)^2$	write the trinomial as a perfect square
$\left(x + \frac{b}{2a}\right)^2 = -\frac{c}{a} \cdot \frac{4a}{4a} + \frac{b^2}{(2a)^2}$	multiply by $-\frac{c}{a}$ to get like denominators
$\left(x + \frac{b}{2a}\right)^2 = -\frac{4ac}{4a^2} + \frac{b^2}{4a^2}$	simplify $\left(\frac{b}{2a}\right)^2$
$\left(x + \frac{b}{2a}\right)^2 = \frac{-4ac + b^2}{4a^2}$	combine fractions with like denominators
$\sqrt{\left(x + \frac{b}{2a}\right)^2} = \sqrt{\frac{b^2 - 4ac}{4a^2}}$	take the square root of both sides
$x + \frac{b}{2a} = \pm\frac{\sqrt{b^2 - 4ac}}{2a}$	simplify ($\sqrt{4a^2} = 2a$)
$x = -\frac{b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{2a}$	subtract $\frac{b}{2a}$ from both sides
$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$	combine fractions with like denominators

Answers:

	Algebra	Step
1.	$-x^2 + 9x - 16 = 0$	original problem
2.	$\frac{-x^2}{-1} + \frac{9x}{-1} - \frac{16}{-1} = \frac{0}{-1}$	divide both sides by $a = -1$

3.	$x^2 - 9x + 16 = 0$	simplify
4.	$x^2 - 9x = -16$	add 15 to both sides
5.	$x^2 - 9x + 20.25 = -16 + 20.25$	add $\left(\frac{-9}{2}\right)^2 = (4.5)^2 = 20.25$ to both sides
6.	$x^2 - 9x + 20.25 = 4.25$	simplify
7.	$(x - 4.5)^2 = 4.25$	write the trinomial as a perfect square
8.	$(x - 4.5)^2 - 4.25 = 0$	set one side equal to 0
9.	$(x - 4.5)^2 - 4.25 = 0$	starting equation
10.	$(x - 4.5)^2 = 4.25$	add 4 to both sides
11.	$\sqrt{(x-4.5)^2} = \pm\sqrt{4.25}$	take the square root of both sides
12.	$(x - 4.5) = \pm\sqrt{4.25}$	simplify
13.	$x - 4.5 = +\sqrt{4.25}$ or $x - 4.5 = -\sqrt{4.25}$	break into two equations
14.	$x = 4.5 + \sqrt{4.25} \approx 6.56$ or $x = 4.5 - \sqrt{4.25} \approx 2.44$	solve each

15. Where does this bridge section meet ground level?

Answer: (2.44, 0) and (6.56, 0)

16. What is the span of this section (the distance from one ground level point to another)?

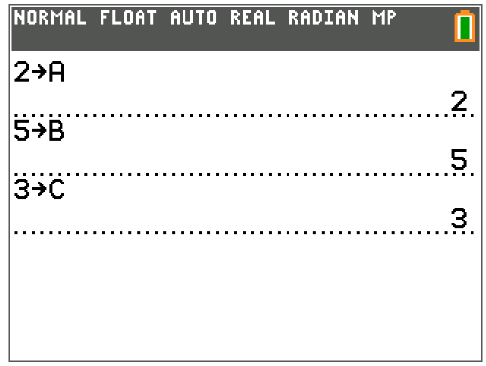
Answer: 4.12 units

17. Rewrite the original equation, $y = -x^2 + 9x - 16$, in the form $y = a(x - h) + k$. What are the corresponding equations for the next two spans of the trestle?

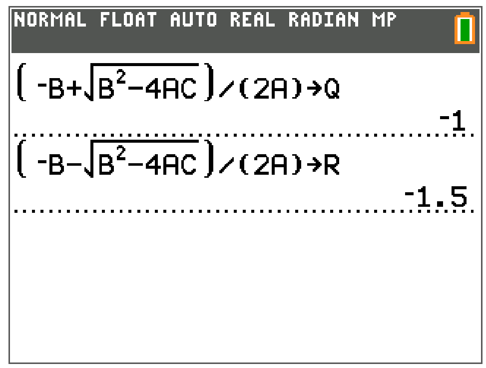
Answers: $y = -(x - 4.5) + 4.25$; $y = -(x - 8.74) + 4.25$; $y = -(x - 12.85) + 4.25$

Problem 2 – Using the Quadratic Formula

Students can use their graphing calculators to calculate the quadratic formula. Students should store the values of **A**, **B**, and **C** to match the equation $2x^2 + 5x + 3 = 0$.



Because of the ± sign in the quadratic equation, it must be stored it in two pieces: **Q**, with a + instead of the ±, and **R**, with a – instead of the ±. Students should define **Q** and **R**. (**Q** is shown.)



<u>OPTIONAL</u>:

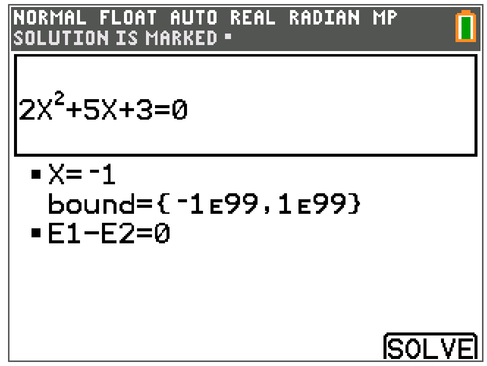
There is another way students can solve quadratic equations with their calculators using the **Numeric Solver**. The **Numeric Solver** tries many different values for the variable until it finds one that works. Open it by going to the **Math** menu and choosing it from the list.

Enter $2x^2 + 5x + 3$ in **E1** and 0 in **E2** and press [graph]. You can guess the solution on the second line and enter an upper and lower bound for the values where the **Numeric Solver** will look for the solution.

Note that the **Numeric Solver** is asking for the same information that a **Calculate** command such as **maximum** asks for on the graph screen.

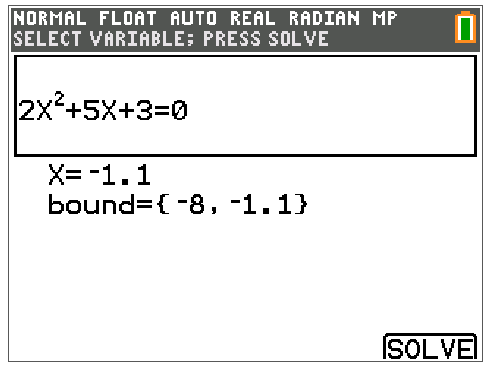
> **Tech Tip:** If using the TI-84 Plus, select **Solver…** from the **Math** menu. Enter the equation $2x^2 + 5x + 3 = 0$ on the first line at the top of the screen. You can guess the solution on the second line and enter an upper and lower bound for the values where the **Numeric Solver** will look for the solution. Press [alpha] [enter] to run the command.

Press [enter] [graph] to run the command. You will notice that the **Numeric Solver** returns only one solution, even though the equation has two solutions. This is because the **Numeric Solver** stops looking once it finds a value of the variable that makes the equation true. (This solution also may not be exact.)



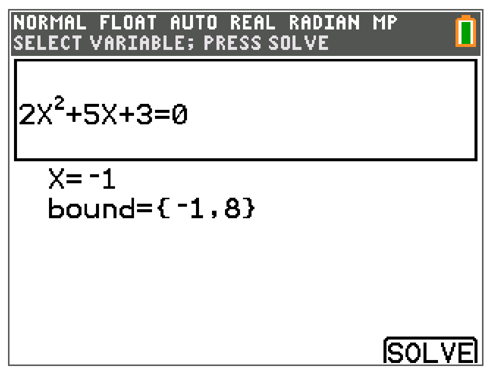
The expression **E1–E2=0** means that the **Numeric Solver** has checked the solution by substituting it into both sides of the equations and then subtracting the right side from the left, much as you would check the answer to an equation!

To find both solutions, you must run the **Numeric Solver** twice and tell it where to look for the solutions, as in the screens shown.



It is not always easy to guess where to tell the **Numeric Solver** to look for the solution. For example, if you had looked at < 0 and ≥ 0, you would not have found both solutions to this equation.

The quadratic formula is usually a better tool for solving quadratic equations with your calculator.



Answers:

18. $-55x + 30 = 50x^2$

$x = -\frac{3}{2}$; $x = \frac{2}{5}$

19. $x^2 + 2x + 1 = 0$

$x = -1$

20. $6x^2 + x = 12$

$x = \frac{4}{3}$; $x = -\frac{3}{2}$

21. $3x^2 = 2x + 5$

$x = \frac{5}{3}$; $x = -1$

22. $-11x^2 + 4x + 7 = 0$

$x = -\frac{7}{11}$; $x = 1$

23. $-4x^2 + 16x = -28$

$x \approx -1.317$; $x \approx 5.317$

24. $2x^2 = -9x - 4$

$x = -\frac{1}{2}$; $x = -4$

25. $3x^2 + 8x - 11 = 0$

$x = 1$; $x = -\frac{11}{3}$

26. $-2x^2 - 5x + 9 = 0$

$x \approx -3.712$; $x \approx 1.212$

27. Graph the function $y = x^2 + 4x + 4$. Solve the corresponding equation $x^2 + 4x + 4 = 0$ using the different methods you've learned (completing the square, using the quadratic formula, etc.). Explain how you got the results, and how the results relate to the graph of the function.

Answer: There is only one solution ($x = -2$) because the term in the square root is zero. Graphically, this corresponds to the fact that the curve touches the x-axis at only one point.

28. Graph the function $y = x^2 + 4x + 6$. Solve the corresponding equation $x^2 + 4x + 6 = 0$ using the different methods you've learned (completing the square, using the quadratic formula, etc.). Explain how you got the results, and how the results relate to the graph of the function.

Answer: There are no real solutions because term in the square root is negative, and the square root of a negative number is not real. Graphically this corresponds to the fact that the curve does not touch the x-axis anywhere.