Hypothesis Testing: Means

ID: 10239

Time required
40 minutes

Activity Overview

Students test a claim about a mean with a large sample size at the 5% significance level. They find the test statistic and compare it to the critical value. They also find the area under the curve to find the P-value. Then, they see how the result would change if a 1% significance level were used instead. In the second half of the activity, students suppose that the sample mean and standard deviation came from a smaller sample size and see how that change reflects in the results of the hypothesis test. An optional extension further allows students to study the importance of sample size.

Topic: Hypothesis Testing

- *Use the sampling distribution of the mean $\bar{x}$ of the population (with known standard deviation σ) to test the null hypothesis H_0: $\mu = \mu_0$ against the alternative one-tailed hypothesis H_a: $\mu < \mu_0$ or the two-tailed hypothesis H_a: $\mu = \mu_0$.*

Teacher Preparation

- *This activity can be used while introducing hypothesis testing, to compare methods of hypothesis testing (critical value or P-value), or to compare testing claims about a mean with large and small sample sizes.*
- *This activity assumes familiarity with the normal distribution and finding a z-score, $z = \dfrac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}}$, and the t-distribution and finding a t-score, $t = \dfrac{\bar{x} - \mu}{\frac{s}{\sqrt{n}}}$.*
  For large n, s approximates σ.
- *Information for an optional extension is provided at the end of the activity on the student worksheet. Should you not wish students to complete the extension, have students disregard that portion of the student worksheet.*
- ***To download the student worksheet, go to education.ti.com/exchange and enter "10239" in the quick search box.***

Associated Materials

- *HypothesisTestingMeans_Student.doc*

Suggested Related Activities

To download any activity listed, go to education.ti.com/exchange and enter the number in the keyword search box.

- *Cancer Clusters (TI-Nspire technology) — 9996*
- *Run Me a Hypothesis Test (TI-84 Plus family with TI-Navigator) — 5135*
- *Candy Pieces (TI-84 Plus family) — 10039*
- *Testing Claims About Proportions (TI-84 Plus family) — 10130*

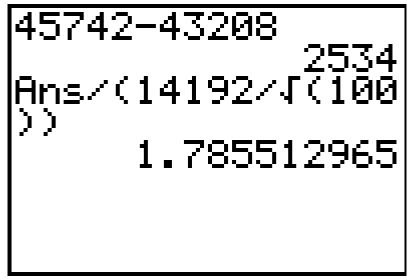
Problem 1 – Large Sample, α = 0.05

Students are introduced to the scenario of a community will a mean salary of $43,208. They are to determine the null and alternative hypotheses.

This is a right-tailed test so, H_0: μ = $43,208, H_a: μ > $43,208.

Discuss whether to use a *z*-score (the normal distribution) or a *t*-score (the *t* distribution) for the test statistic. Students should say a *z*-score should be used because *n* is large.

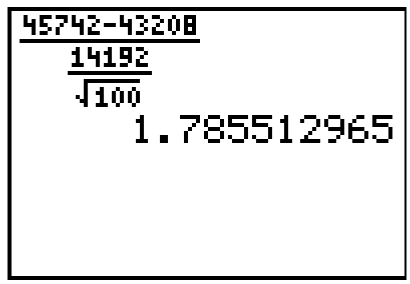
The sample mean and standard deviation from the 100 residents are $\bar{x} = \$45{,}742$ and *s* = $14,192.



Students are to use this information to calculate the test statistic. Because *n* is large, the sample standard deviation is a good estimate of the population standard deviation.

If using Mathprint OS:

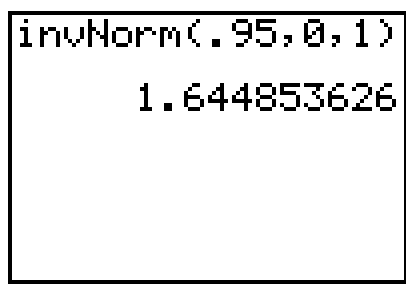
Students can display the formula as a fraction. To do this, press [ALPHA] [F1] and select **n/d**. Then enter the value of the numerator, press [▼] to move to the denominator. Press [ALPHA] [F1] and select **n/d**. again to create a fraction within the denominator. Enter the value of the denominator and press [ENTER] to evaluate.



Press [▶] to move out of the fraction. Also, parentheses are not needed around the numerator or denominator.

Here, *z* = 1.78551. Have students state what this represents. (It is the number of standard deviations that $45,742 is from $43,208.) The question for students to think about is, *Is this a reasonable number of standard deviations for a sample mean to be from zero if the true mean is $43,208?* The critical value will determine that.

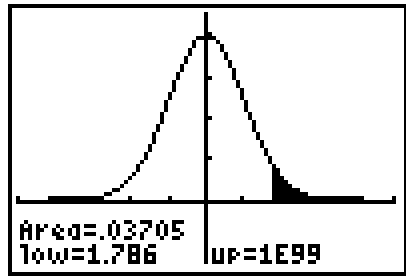
Now students are to find the critical value for α = 0.05. Remind them that this is a right-tailed test, so 5% is above the critical value, 95% is below.



Students can find the critical value by using the **invNorm** command from the DISTR menu.

Ask students if they should reject or fail to reject the null hypothesis and why. (Because the test statistic is greater than the critical value, they should reject the null hypothesis.)

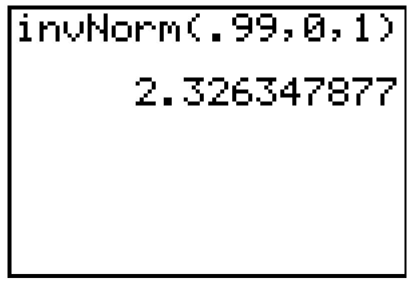
Students should make sure that all functions and plots have been turned off.



Now, students are to find the *P*-value. Before using the **ShadeNorm** command, students will need to use the **ClrDraw** command from the DRAW menu ([2nd] [DRAW]).

The area is about 0.03705. The value is not doubled because it is a one-tailed test. Because it is less than the significance level of 0.05, it was correct to reject the null hypothesis.

Problem 2 – Large Sample, α = 0.01



Students are to test the hypothesis again, this time with α = 0.01.

The test statistic, 1.78551, will remain the same because the sample size did not change; however, a new critical value needs to be calculated.

Discuss with students if they should reject or fail to reject the null hypothesis and why. They should fail to reject the null hypothesis because the test statistic is not in the critical region, that is, the region beyond the critical value.

Be sure students understand what the critical value stands for. In this case, to reject the null hypothesis, the sample mean would have to be greater than 2.32635 standard deviations above the mean.

Discuss how the null hypotheses is not rejected with α = 0.01. A *P*-value < 0.01 is more significant than that of one between 0.01 and 0.05, requiring the sample mean to be even more standard deviations away from the claimed mean.

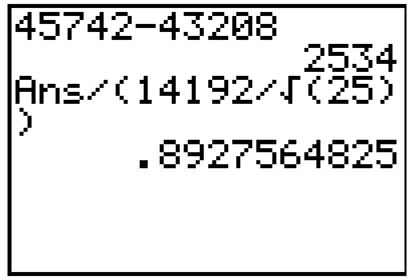
Explain that they have shown what happens with both significance levels for educational purposes. In a real-life application, the significance level is chosen before the hypothesis test is performed. It remains the same throughout the test, and is not changed once the *P*-value is known. If the *P*-value is close to the significance level, more samples and tests may be employed.

Problem 3 – Small Sample, α = 0.05

To see how sample size affects the results, students suppose that n = 25 rather than n = 100 and see how sample size affects the results.

Ask students if they would find a *z*-score or *t*-score and why. (Because the sample size is small and the population standard deviation is unknown, the *t*-distribution is used.) They should assume that the population has a roughly normal distribution.

Students are to calculate the test statistic.
(t = 0.892756)

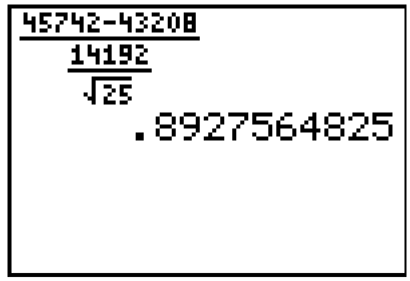


They may see that this does not seem very far from the mean, but they'll need to compare it to the critical value before making judgment. The t-distribution has different critical values than the normal distribution for small values of n.

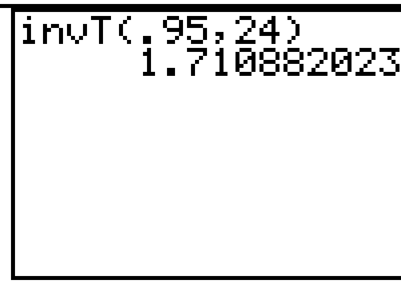
If using Mathprint OS:

Remind students to display the formula as a fraction. This will help students prevent errors.

Press ▶ to move out of the fraction or from below the radical sign.

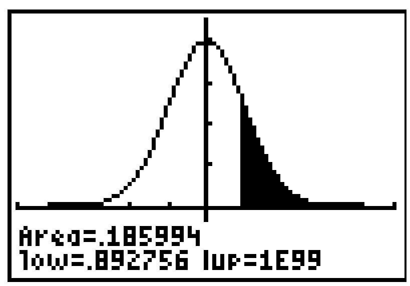


Students will then calculate the critical value. Remind them that there are n – 1 degrees of freedom and that the test is right-tailed.



The test statistic is not greater than the critical value. The evidence is not strong enough to reject the null hypothesis.

Students are to find the P-value using the **ShadeNorm** command. As should be expected, the P-value is much greater than 0.05.



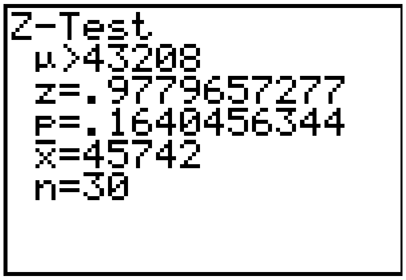
Students are to discuss why they fail to reject the null hypothesis.

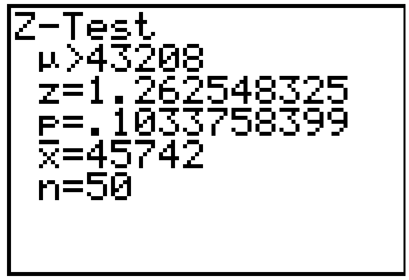
Remind them that when n = 100, we did reject the null hypothesis at this same significance level. Ask them why the sample size would change this result. You may wish for them to complete the extension before answering this.

Problem 4 – Extension

Students can use the **Z-Test** or **T-Test** commands from the TESTS menu to find the test statistic and P-value for sample sizes between 25 and 100. Discuss how these values change and why.

As n increases, the test statistic increases and the P-value decreases. As the sample size increases, sample means are closer to the true mean (Central Limit Theorem), and the evidence does not have to be as extreme as with smaller samples to warrant a rejection of the null hypothesis.





Solutions

1. H_0: μ = \$43,208, H_a: μ > \$43,208

2. A z-score because the sample size is large.

3. about 1.786

4. about 1.645

5. Reject; The test statistic is greater than the critical value and it is in the critical region.

6. about 0.03705

7. about 0.03705

8. It is less than 5%, so it was correct to reject the null hypothesis.

9. about 2.326

10. Fail to reject; The test statistic is less than the critical value and is not in the critical region. Alternatively, one could say that the P-value is greater than 0.01.

11. t-score; Because the sample size is small and the population standard deviation is unknown, the t-distribution is used.

12. about 0.893

13. about 1.711

14. Fail to reject; The test statistic is not greater than the critical value.

15. It must be greater than 0.05 because that is when we fail to reject the null hypothesis.

16. about 0.19